This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

Educ T 118.65.339

Harvard College Library

FROM

Miss Anna Barrows

3 2044 096 997 168

Educ T 118.65.339

Harvard College Library

FROM

Miss Anna Barrows

3 2044 096 997 168

A

KEY OF SOLUTIONS

TO EXAMPLES IN

EATON'S HIGH SCHOOL ARITHMETIC.

BY

JAMES S. EATON, M.A.,
INSTRUCTOR IN PHILLIPS ACADEMY, ANDOVER, AND AUTHOR OF A
SERIES OF ARITHMETICS.

BOSTON:
TAGGARD AND THOMPSON,
20 CORNHILL.
SAN FRANCISCO: H. H. BANCROFT & COMPANY.
1865.

Educ T 118.65.339

HARVARD COLLEGE
4 Oct 1929
LIBRARY

Miss Anna Barrows

Entered, according to Act of Congress, in the year 1865, by
JAMES S. EATON, M.A.,
In the Clerk's Office of the District Court of the District of Massachusetts.

Stereotyped and Printed by W. F. Draper,
Andover, Mass.

PREFACE.

HAVING received repeated and earnest solicitations from various teachers to furnish entire manuscript solutions of the examples in his High School Arithmetic, and numerous requests from all parts of the country for solutions of particular examples, the author has been constrained to issue this Key, not only in self-defense and in a spirit of kindness to that class of teachers who desire a Key, but from a conviction, induced by the nature of the many letters received asking for assistance, that the presentation of good modes of solution and explanation may suggest valuable ideas to many teachers, and through them benefit the pupils who are using the Arithmetic.

There are various methods of solving many of the examples. In every case much care has been exercised to adopt a good, clear, sensible solution; and, in many cases, two or three methods have been given, each having some advantages over the others, or one being adapted to one class of minds and another to another class.

Uniformity has not been sought, but variety rather; the design being to make the Key both plain and suggestive. A few diagrams have been inserted as furnishing the best suggestions that can be given for the solution of certain examples, and the teacher will readily draw upon the blackboard many other figures to facilitate the solution of other examples.

In many of the examples, for the sake of brevity, the numbers are

Educ T 118.65.339

Harvard College Library

FROM

Miss Anna Barrows

3 2044 096 997 168

MULTIPLICATION.

4.	(p. 21.) 4,099,596,898
5.	219,478,478,003
6.	3,593,658,182,596,616
7.	330,823,998
8.	410,321,203,491,363,739,530
10.	184,538,719,411
11.	780,541,608,003
14.	8,987,796,249
15.	220,133,459,476
16.	121,932,631,112,635,269
17.	99,999,999,980,000,000,001
18.	11,111,108,888,889
19.	3,703,703,629,629,630
20.	248,404,659,240,286,080
21.	369,120,793,361,690,448
22.	78,271,880,680,185,984
23.	4,406,110,361,509,205,524,506
24.	485,313,716,210,410,832,971,116
25.	63,601,595,294,585,507,822,208
29.	194,152
30.	45,306
31.	566,568
32.	38,059,956
33.	827,658,432
34.	517,197,024
35.	48,385,638
38.	4,698,000
40.	59,874,000,000,000
44.	63,139,910,400,000
45.	4,639,527,515,000,000
49.	4,377,317,000,000
50.	427,071,338,400,000
53.	293,343,530,562
54.	227,797,569,155,574
55.	1,566,286,641,827,600,000
58.	4,725
59.	432,000
61.	$57,987
62.	817m.
63.	7,336m.
64.	77,486
66.	95,400
67.	$11,595
68.	$9,724

DIVISION.

		Quotient.	Rem.
6.	(p. 28.)	52,954	3
8.		14,921	
9.		17,539,085	
10.		715,406,265	8
13.		5,086	
15.		12,654	2
16.		20,160	4,444
17.		99,891,686	2,955

	Quotient.	Rem.		Quotient.	Rem.
20.	757		49.	2,010	7,080
21.	856		50.	10,865	25,821
22.	345		51.	10,865	258,210
26.	6,541		52.	1,358	14,589
33.	751	70	54.	476	
35.	686	259	55.	888	
36.	6	101	56.	964	
37.	8	2,475	57.	15,608	5
39.	40,276	3	58.	5,502	
42.	30	78,654,321	59.	89	
44.	28	275	60.		12
46.	83	3,087	61.	128	
48.	300	9,999	62.	1,728	

REDUCTION.

(Art. 87. p. 46.)

(2.) 7l.m. 3wk. 5d. 19h. 53m. 45sec.
4
31wk.
7
222d.
24
888
444
19
5347h.
60
320873m.
60
19252425sec., Ans.

Note. — The several numbers of the lower denominations may be added *mentally, writing only the results*, as, in this operation, is done with the 3wk., 5d., 53m., and 45sec.; or, in some cases, it may be easier, and ∴ more convenient, to set the number *under* the partial product, and then add, as is here done with the 19h.

(5.) 67t. 17cwt. 2qr. 23lb. 14oz. 11dr.
20
1357cwt
4
5430qr.
25
27150
10860
23
135773lb.
16
814638
135773
14
2172382oz.
16
13034292
2172382
11
34758123dr., Ans.

(4.) 4c.yd. 24cu.ft. 1695c.in.
27
28
8
24
132cu.ft.
1728
1056
264
924
132
1695
229791c.in., Ans.

(3.) 53bush. 3pk. 7qt. 1pt.
4
215pk.
8
1727qt.
2
3455pt., Ans.

(8.) 87yd. 3qr. 2na.
4
351qr.
4
1406na., Ans.

(6.) 17lb. 10oz. 7dr. 2sc. 19gr.
12
214oz.
8
1719dr.
3
5159sc.
20
103199gr., Ans.

(7.) 273lb. 11oz. 19dwt. 21gr.
12
3287oz.
20
65759dwt.
24
263036
131518
21
1578237gr., Ans.

(9.) 69m. 5fur. 37rd.
8
557fur.
40
22317rd., Ans.

(11.) 7m. 7fur. 9ch. 3rd. 21 li.
8
63fur.
10
639ch.
4
2559rd.
25
12795
5118
21
63996 li., Ans.

(10.) 5yd. 2ft. 7in. 2b.c.
3
17ft.
12
211in.
3
635b.c., Ans.

(12.) 37sq.m. 637a. 3r. 37sq.rd.
640
1480
222
637
24317a.
4
97271r.
40
3890877sq. rd., Ans.

(13.) 14c. 7c.ft. 15cu.ft. 1716c.in.
8
119c.ft.
16
714
119
15
1919cu.ft.
1728
15352
3838
13433
1919
1716
3317748c.in., Ans.

(14.) 15gal. 2qt. 1pt. 3gi.
4
62qt.
2
125pt.
4
503gi., Ans.

(15.) 14circ. 7s. 17° 57′ 14″
12
175s.
30
5267°
60
316077′
60
18964634″, Ans.

(16.) 6 l.m. 2wk. 0d. 18h. 0m. 47sec.
4
26wk.
7
182d.
24
728
364
18
4386h.
60
263160m.
60
15789647sec., Ans.

(17.) 4m. 0fur. 8ch. 2rd.
8
32fur.
10
328ch.
4
1314rd.
25
6570
2628
32850li., Ans.

Note. — Instead of multiplying by 10 and adding 8, as in the operation, the same result is more easily found by *annexing* 8 to the 32.

(18.) 14t. 24lb.
2000
28024lb.
16
168144
28024
448384oz., Ans.

(Art. **89**, p. 47.)

(3.) 2)3455pt.
8)1727qt. 1pt.
4)215pk. 7qt.
53bush. 3pk.
Ans. 53bush. 3pk. 7qt. 1pt.

(4.) 1728)229791c.in.
27)132cu.ft. 1695c.in.
4c.yd. 24cu.ft.
Ans. 4c.yd. 24cu.ft. 1695c.in.

Note. — When the divisor is large, the work is done by Long Division, on the slate or elsewhere, only the result being written here.

(5.) 16)34758123dr.
16)2172382oz. 11dr.
25)135773lb. 14oz.
4)5430qr. 23lb.
20)1357cwt. 2qr.
67t. 17cwt.
Ans. 67t. 17cwt. 2qr. 23lb. 14oz. 11dr.

(6.) 20)103199gr.
3)5159sc. 19gr.
8)1719dr. 2sc.
12)214oz. 7dr.
17 lb. 10oz.
Ans. 17 lb. 10oz. 7dr. 2sc. 19gr.

(7.) 24)1578237gr.
20)65759dwt. 21gr.
12)3287oz. 19dwt.
273 lb. 11oz.
Ans. 273 lb. 11oz. 19dwt. 21gr.

(8.) 4)1406na.
4)351qr. 2na.
87yd. 3qr.
Ans. 87yd. 3qr. 2na.

(9.) 40)22317rd.
8)557fur. 37rd.
69m. 5fur.
Ans. 69m. 5fur. 37rd.

(10.) 3)635b.c.
12)211in. 2b.c.
3)17ft. 7in.
5yd. 2ft.
Ans. 5yd. 2ft. 7in. 2b.c.

(11.) 25)63996 li.
4)2559rd. 21 li.
10)639ch. 3rd.
8)63fur. 9ch.
7m. 7fur.
Ans. 7m. 7fur. 9ch. 3rd. 21 li.

(13.) 1728)3317748c.in.
16)1919cu.ft. 1716c.in.
8)119c.ft. 15cu.ft.
14c. 7c.ft.
Ans. 14c. 7c.ft. 15cu.ft. 1716c.in.

(12.) 40)3890877sq.rd.
4)97271r. 37sq.rd.
640)24317a. 3r.
37sq.m. 637a.
Ans. 37sq.m. 637a. 3r. 37sq.rd.

(14.) 4)503gi.
2)125pt. 3gi.
4)62qt. 1pt.
15gal. 2qt.
Ans. 15gal. 2qt. 1pt. 3gi.

(15.) 60)18964634″
60)316077′ 14″
30)5267° 57′
12)175s. 17°
14circ. 7s.

Ans. 14circ. 7s. 17° 57′ 14″

(16.) 60)15789647sec.
60)263160m. 47sec.
24)4386h.
7)182d. 18h.
4)26wk.
6 l.m. 2wk.

Ans. 6 l.m. 2wk. 18h. 47sec.

(17.) 25)32850 li.
4)1314rd.
10)328ch. 2rd.
8)32fur. 8ch.
4m.

Ans. 4m. 8ch. 2rd.

(18.) 16)448384oz.
25)28024lb.
4)1120qr. 24 lb.
20)280cwt.
14t.

Ans. 14t. 24 lb.

GENERAL PRINCIPLES.

(ART. **108**, p. 54.)

(5.) 2)242550
3)121275
3)40425
5)13475
5)2695
7)539
7)77
11

Ans. 2, 3, 3, 5, 5, 7, 7, 11.

(6.) 2)1801800
2)900900
2)450450
3)225225
3)75075
5)25025
5)5005
7)1001
11)143
13

Ans. 2, 2, 2, 3, 3, 5, 5, 7, 11, 13.

(ART. **109.**)

(3.) 1, 2, 4, 8, 16, 3, 6, 12, 24, 48, 9, 18, 36, 72, 144.
(4.) 1, 5, 25, 125, 2, 10, 50, 250, 4, 20, 100, 500.

(ART. **110—113**, p. 56.)

5. 21 6. 24

8.

	Quotients.	39912
27088	× 1 =	27088
25648	= 2 ×	12824
1440	× 8 =	11520
1304	= 1 ×	1304
136	× 9 =	1224
80	= 1 ×	80
56	× 1 =	56
48	= 2 ×	24
Ans. 8	× 3 =	24
		0

NOTE.— This operation is more *compact* than that given in the Arithmetic for Ex. 7, but the reasoning is *precisely the same.* The skillful teacher will abbreviate still more, by *omitting the rule marks and the column of quotients, carrying the quotients in the mind as he explains.* See the solution of Ex. 9.

(9.)		2018835	(11.)		437437
	437437	1749748		9360	430560
	269087	269087		6877	6877
	168350	168350		2483	4966
	100737	100737		1911	1911
	67613	67613		572	1716
	66248	33124		390	195
	1365	32760		182	182
	1092	364		182	Ans. 13
	273	273		0	
	273	Ans. 91			
	0				

By the above process 13 is found to be the greatest common divisor of 9360 and 437437, and, by trial, 13 is also found to be a divisor of 2018835; ∴ 13 is the greatest common divisor of the three given numbers.

12. 27
14. 1
15. 1

(Art. 114, p. 59.)

2.
$$\begin{aligned} 6 &= 2 \times 3 \\ 8 &= 2 \times 2 \times 2 \\ 12 &= 2 \times 2 \times 3 \\ 18 &= 2 \times 3 \times 3 \\ 24 &= 2 \times 2 \times 2 \times 3 \end{aligned}$$

Ans. $2^3 \times 3^2 = 72$.

3.
$$\begin{aligned} 48 &= 2 \times 2 \times 2 \times 2 \times 3 \\ 96 &= 2 \times 2 \times 2 \times 2 \times 2 \times 3 \\ 144 &= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \\ 192 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \end{aligned}$$

Ans. $2^6 \times 3^2 = 576$.

4. 660
5. 2,160
6. 4,032
9. 2,042,040

(10.)
$$\begin{array}{r} 5)\underline{\cancel{1}\cancel{0},\ \cancel{1}\cancel{5},\ \cancel{4}\cancel{5},\ 75,\ 90} \\ 3)\underline{15,\ 18} \\ 5,\ \ 6 \end{array}$$

Ans. $5 \times 3 \times 5 \times 6 = 450$.

11.
$$\begin{array}{r} 2)\underline{\cancel{4},\ 6,\ 8,\ 9} \\ 3)\underline{3,\ 4,\ 9} \\ 1,\ 4,\ 3 \end{array}$$

Ans. $2 \times 3 \times 4 \times 3 = 72$.

12. $\cancel{8},\ \cancel{1}\cancel{2},\ \cancel{1}\cancel{6},\ \cancel{2}\cancel{4},\ \cancel{3}\cancel{2},\ \cancel{4}\cancel{8},\ 96$ Ans. 96.

13. $\cancel{8}\cancel{0},\ \cancel{2}\cancel{0},\ \cancel{1}\cancel{6}\cancel{0},\ \cancel{4}\cancel{0},\ \cancel{5},\ 320,\ \cancel{1}\cancel{0},\ \cancel{1}\cancel{6}$ Ans. 320.

14.
$$\begin{array}{r} 13)\underline{91,\ 3523,\ 6487} \\ 7,\ \ 271,\ \ 499 \end{array}$$

Ans. $13 \times 7 \times 271 \times 499 = 12{,}305{,}839$.

15.
$$\begin{array}{r} 7)\underline{12089,\ 1309,\ 2849,\ 2233} \\ 11)\underline{1727,\ \ 187,\ \ 407,\ \ 319} \\ 157,\ \ \ 17,\ \ \ 37,\ \ \ 29 \end{array}$$

Ans. $7 \times 11 \times 157 \times 17 \times 37 \times 29 = 220{,}515{,}449$.

(16.)
$$\begin{array}{r} 7\overline{)\cancel{28},\ 42,\ 56,\ 70,\ 80,\ 90} \\ 2\overline{)\cancel{6},\ \cancel{8},\ \cancel{10},\ 80,\ 90} \\ 5\overline{)40,\ 45} \\ 8,\ \ 9 \end{array}$$

Ans. $7 \times 2 \times 5 \times 8 \times 9 = 5040$.

17. $8 \times 15 \times 77 = 9240$, Ans.
18. $1181 \times 2741 \times 3413 = 11{,}048{,}293{,}973$, Ans.
20. $35 \times 49 \div 7 = 245$, Ans.
21. $39 \times 91 \div 13 = 273$, Ans.

FRACTIONS.

(Art. **125**, p. 65.)

5. $\frac{897}{27}$ 16. $\frac{23}{36} \times 7 \times 6 = \frac{161}{6}$, Ans.
6. $\frac{326}{693}$ 17. $\frac{73}{108} \times 7 \times 12 = \frac{511}{9}$, Ans.
10. $\frac{13512}{3391}$ 18. $\frac{156}{33} \times 4 \times 11 = \frac{624}{3}$, Ans.
11. $\frac{10582}{210}$ 19. $\frac{78569}{99} \times 7 \times 9 = \frac{549983}{11}$, Ans.
12. $\frac{26833022}{3570}$ 20. $\frac{278}{1728} \times 7 \times 144 = \frac{1946}{12}$, Ans.
23. 174 | 24. 73 | 25. 74698

(Art. **126**, p. 67.)

3. $\frac{4}{796}$
4. $\frac{5}{469}$ 16. $\frac{161}{6} \div (7 \times 6) = \frac{23}{36}$, Ans.
5. $\frac{7}{746}$ 17. $\frac{511}{9} \div (7 \times 12) = \frac{73}{108}$, Ans.
6. $\frac{27}{1987}$
9. $\frac{87}{780}$ 18. $\frac{624}{3} \div (4 \times 11) = \frac{156}{33}$, Ans.
10. $\frac{1667}{675864}$ 19. $\frac{549983}{11} \div (7 \times 9) = \frac{78569}{99}$, Ans.
11. $\frac{3391}{3404}$ 20. $\frac{1946}{12} \div (14 \times 72) = \frac{139}{864}$, Ans.
12. $\frac{47}{646380}$

(Art. **127**, p. 69.)

3. $\frac{391}{406}$ | 4. $\frac{126}{4147}$ | 5. $\frac{161}{75}$

9. $\frac{\overset{3}{\cancel{75}}}{\underset{47}{\cancel{94}}} \times \frac{\overset{4}{\cancel{8}}}{\underset{5}{\cancel{125}}} = \frac{12}{235}$, Ans. 10. $\frac{\cancel{18}}{\underset{5}{\cancel{35}}} \times \frac{\overset{3}{\cancel{21}}}{\underset{11}{\cancel{198}}} = \frac{3}{55}$, Ans.

11. $\frac{\overset{7}{\cancel{77}}}{\underset{6}{\cancel{108}}} \times \frac{\cancel{18}}{\underset{5}{\cancel{55}}} = \frac{7}{30}$, Ans. 12. $\frac{\overset{8}{\cancel{64}}}{\underset{21}{\cancel{315}}} \times \frac{\overset{5}{\cancel{75}}}{\underset{7}{\cancel{56}}} = \frac{40}{147}$, Ans.

16. $\frac{\cancel{1728} \times \overset{1}{\cancel{893}}}{\underset{5}{\cancel{4465}} \times \underset{3}{\cancel{5184}}} = \frac{1}{15}$, Ans.

17. $\frac{\cancel{698} \times \overset{1}{\cancel{347}}}{\underset{7}{\cancel{2429}} \times \underset{7}{\cancel{4886}}} = \frac{1}{49}$, Ans.

18. $\frac{\overset{2}{\cancel{976}} \times \overset{5}{\cancel{2135}}}{\cancel{427} \times \cancel{488}} = 10$, Ans. 19. $\frac{\overset{4}{\cancel{84}} \times \overset{2}{\cancel{190}}}{\cancel{95} \times \cancel{21}} = 8$, Ans.

20. $\frac{\overset{8}{\cancel{876968}} \times \overset{11}{\cancel{5236}}}{\cancel{476} \times \cancel{109621}} = 88$, Ans.

26. $\frac{\cancel{3} \times \overset{2}{\cancel{10}} \times \cancel{4} \times \cancel{7} \times \cancel{11}}{\cancel{5} \times \underset{\cancel{4}}{\cancel{12}} \times 9 \times \cancel{11} \times \underset{3}{\cancel{21}}} = \frac{2}{27}$, Ans.

27. $\frac{\cancel{4} \times \overset{\cancel{5}}{\cancel{45}} \times \cancel{19} \times \overset{\cancel{4}}{\cancel{28}} \times \overset{2}{\cancel{14}}}{\cancel{9} \times \underset{\underset{5}{\cancel{20}}}{\cancel{80}} \times \underset{\cancel{7}}{\cancel{35}} \times \underset{3}{\cancel{57}} \times \underset{7}{\cancel{49}}} = \frac{2}{105}$, Ans.

28. $\frac{\overset{\cancel{11}}{\cancel{55}} \times \overset{3}{\cancel{51}} \times \cancel{9} \times \overset{3}{\cancel{24}}}{\cancel{17} \times \underset{\cancel{9}}{\cancel{99}} \times \cancel{5} \times \cancel{8}} = 9$, Ans.

29. $\frac{\cancel{8} \times \overset{2}{\cancel{34}} \times \overset{9}{\cancel{81}} \times \overset{\cancel{12}}{\cancel{1728}}}{\cancel{17} \times \cancel{9} \times \underset{\cancel{8}}{\cancel{96}} \times \cancel{144}} = 18$, Ans.

31. $\frac{\overset{7}{\cancel{35}} \times 3}{\cancel{5}} = 21$, Ans.   32. $\frac{\overset{3}{\cancel{48}} \times 5}{\cancel{16}} = 15$, Ans.

33. $\frac{\overset{8}{\cancel{64}} \times 7}{\underset{3}{\cancel{24}}} = \frac{56}{3}$, Ans.   34. $\frac{\cancel{1056} \times \overset{373}{\cancel{746}}}{\underset{\underset{2}{\cancel{4}}}{\cancel{4224}}} = \frac{373}{2}$, Ans.

(ART. **128**, p. 72.)

3. $\frac{559}{464}$ 4. $\frac{247}{1113}$ 5. $\frac{931}{425}$ 6. $\frac{11869}{4503}$

8. $\frac{7 \times 16}{8 \times 19} \div \frac{4 \times 33}{11 \times 29} = \frac{7 \times \overset{2}{\cancel{16}} \times \cancel{11} \times 29}{\cancel{8} \times 19 \times \underset{2}{\cancel{4}} \times \underset{3}{\cancel{33}}} = \frac{203}{114}$, Ans.

9. $\frac{6 \times 5 \times 3}{11 \times 8 \times 15} \div \frac{4 \times 9}{7 \times 11} = \frac{\cancel{6} \times \cancel{5} \times \cancel{3} \times 7 \times \cancel{11}}{\cancel{11} \times 8 \times \cancel{15} \times \underset{2}{\cancel{4}} \times \underset{3}{\cancel{9}}}$
$= \frac{7}{48}$, Ans.

10. $\frac{16 \times 7}{48 \times 15} \div \frac{11 \times 9 \times 18}{13 \times 4 \times 27} = \frac{\cancel{16} \times 7 \times 13 \times \overset{2}{\cancel{4}} \times \cancel{27}}{\underset{\cancel{3}}{\cancel{48}} \times 15 \times 11 \times \cancel{9} \times \underset{9}{\cancel{18}}}$
$= \frac{182}{1485}$, Ans.

15. $\frac{46}{2} = 23$, Ans.   16. $\frac{7}{4}$, Ans.   17. $\frac{53}{17}$, Ans.

19. $\frac{964}{1563} \div \frac{2}{3} = \frac{482}{521}$, Ans.   20. $\frac{96}{1728} \div \frac{24}{144} = \frac{4}{12}$, Ans.

(ART. **129**, p. 74.)

3. $735)\frac{3675 \mid 5}{6615 \mid 9}$, Ans.   4. $247)\frac{37297 \mid 151}{90649 \mid 367}$, Ans.

5. $3)\frac{2401329 \mid 800443}{2403051 \mid 801017}$, Ans.

6. $945)\frac{17955 \mid 19}{84965 \mid 37}$, Ans. 7. $72)\frac{360 \mid 5}{3024 \mid 42}$, Ans.

8. $5)\frac{17955 \mid 3591}{84965 \mid 16993}$, Ans. 9. $210)\frac{210 \mid 1}{6300 \mid 30}$, Ans.

10. $10)\frac{150 \mid 15}{3640 \mid 364}$, Ans. 11. $1300)\frac{11700 \mid 9}{10400 \mid 8}$, Ans.

12. $2394)\frac{2394 \mid 1}{28728 \mid 12}$, Ans. 13. $8904)\frac{62328 \mid 7}{44520 \mid 5}$, Ans.

14. $225)\frac{3600 \mid 16}{225 \mid 1}$, Ans. 15. $308448)\frac{308448 \mid 1}{15113952 \mid 49}$, Ans.

(Art. **130**, p. 75.)

7. $19874\frac{4}{5}$
8. 144
9. 563
10. $63\frac{2}{3}$
11. $5132\frac{7}{15}$
12. $503\frac{31}{216}$
13. $323\frac{139}{176}$
14. $202\frac{272}{4327}$

(Art. **131**, p. 76.)

4. $\frac{781}{6}$
7. $\frac{99374}{5}$
8. $\frac{361849}{47}$
9. $\frac{1089981}{157}$
10. $\frac{41490467}{895}$
11. $\frac{476929}{1728}$
12. $\frac{7027828}{1973}$
13. $\frac{152399026}{12345}$
14. $\frac{46090522}{6789}$
17. $\frac{7569}{87}$
18. $\frac{2732340}{372}$
19. $\frac{846}{18}$
20. $\frac{126982}{173}$

(Art. **132**, p. 77.)

2. $\frac{5\frac{7}{12}}{3\frac{5}{11}} = \frac{\frac{67}{12}}{\frac{38}{11}} = \frac{67}{12} \div \frac{38}{11} = \frac{67}{12} \times \frac{11}{38} = \frac{737}{456}$, Ans.

3. $\frac{8\frac{4}{9}}{15\frac{3}{8}} = \frac{\frac{76}{9}}{\frac{123}{8}} = \frac{76}{9} \div \frac{123}{8} = \frac{76}{9} \times \frac{8}{123} = \frac{608}{1107}$, Ans.

4. $\frac{\frac{5}{17}}{8\frac{3}{13}} = \frac{\frac{5}{17}}{\frac{107}{13}} = \frac{5}{17} \div \frac{107}{13} = \frac{5}{17} \times \frac{13}{107} = \frac{65}{1819}$, Ans.

5. $\frac{4\frac{3}{8}}{\frac{2}{8}} = \frac{\frac{35}{8}}{\frac{1}{4}} = \frac{35}{8} \div \frac{1}{4} = \frac{35 \times \cancel{4}}{\cancel{8} \quad 2} = \frac{35}{2}$, Ans.

6. $\dfrac{\frac{1}{2} \times \frac{5}{7} \times 7\frac{3}{8}}{19\frac{6}{25}} = \dfrac{\frac{1}{2} \times \frac{5}{7} \times \frac{59}{8}}{\frac{481}{25}} = \dfrac{1 \times 5 \times 59 \times 25}{2 \times 7 \times 8 \times 481}$

$= \dfrac{7375}{53872}$, Ans.

8. $\dfrac{\frac{7}{19}}{73} = \dfrac{7}{19 \times 73} = \dfrac{7}{1387}$, Ans.

9. $\dfrac{\frac{15}{29}}{82} = \dfrac{15}{29 \times 82} = \dfrac{15}{2378}$, Ans.

11. $\dfrac{19}{\frac{13}{15}} = \dfrac{19 \times 15}{13} = \dfrac{285}{13}$, Ans.

12. $\dfrac{876}{2\frac{1}{4}} = \dfrac{876}{\frac{9}{4}} = \dfrac{876 \times 4}{9} = \dfrac{3504}{9} = 389\frac{1}{3}$, Ans.

13. $\dfrac{18\frac{2}{7}}{\frac{2}{3} \times \frac{3}{5} \times \frac{5}{2}} = \dfrac{18\frac{2}{7} \times \cancel{3} \times \cancel{5} \times \cancel{2}}{\cancel{2} \times \cancel{3} \times \cancel{5}} = 18\frac{2}{7}$, Ans.

14. $\dfrac{\frac{2}{7} \times \frac{5}{3} \times \frac{3}{2} \times \frac{14}{5}}{\frac{3}{11} \times \frac{22}{4} \times \frac{8}{6} \times \frac{1}{2} \times 2} =$

$$\frac{\cancel{2} \times \cancel{5} \times \cancel{3} \times \cancel{14} \times \cancel{11} \times \cancel{4} \times \cancel{6} \times \overset{1}{\cancel{2}}}{\cancel{7} \times \cancel{3} \times \cancel{2} \times \cancel{5} \times \cancel{3} \times \underset{\cancel{2}}{\cancel{22}} \times \underset{\cancel{2}}{\cancel{8}} \times \cancel{2}} = 1, \text{ Ans.}$$

(Art. **133**, p. 79.)

4. $\frac{1760}{3080}, \frac{1400}{3080}, \frac{1232}{3080}, \frac{1155}{3080}$.
5. $\frac{1105}{1326}, \frac{306}{1326}, \frac{546}{1326}$.
6. $\frac{47880}{87780}, \frac{23408}{87780}, \frac{13860}{87780}, \frac{15675}{87780}$.
7. $\frac{1386}{3465}, \frac{990}{3465}, \frac{770}{3465}, \frac{630}{3465}, \frac{6930}{3465}$.
8. $\frac{621}{1656}, \frac{920}{1656}, \frac{504}{1656}$.
9. $\frac{120}{1020}, \frac{1360}{1020}, \frac{255}{1020}, \frac{204}{1020}$.
10. $\frac{360}{840}, \frac{504}{840}, \frac{525}{840}, \frac{560}{840}$.
11. $\frac{3465}{630}, \frac{270}{630}, \frac{280}{630}, \frac{126}{630}$.
12. $\frac{2717}{1672}, \frac{456}{1672}, \frac{1144}{1672}$.
13. $\frac{4085}{13889}, \frac{2193}{13889}, \frac{2261}{13889}$.
14. $\frac{2992275}{5294025}, \frac{3811698}{5294025}, \frac{2745050}{5294025}, \frac{2561625}{5294025}, \frac{962550}{5294025}$.
17. $\frac{18}{24}, \frac{20}{24}, \frac{14}{24}, \frac{9}{24}$.
18. $\frac{60}{140}, \frac{50}{140}, \frac{45}{140}, \frac{16}{140}$.
19. $\frac{63}{462}, \frac{210}{462}, \frac{48}{462}, \frac{49}{462}$.
20. $\frac{96}{180}, \frac{27}{180}, \frac{126}{180}, \frac{44}{180}$.
21. $\frac{18}{102}, \frac{10}{102}, \frac{27}{102}, \frac{15}{102}$.

22. $\frac{420}{540}, \frac{468}{540}, \frac{160}{540}, \frac{75}{540}$.
23. $\frac{180}{270}, \frac{120}{270}, \frac{225}{270}, \frac{36}{270}, \frac{70}{270}$.
24. $\frac{144}{414}, \frac{115}{414}, \frac{153}{414}$.
27. $\frac{4}{12}, \frac{3}{12}, \frac{8}{12}$.
28. $\frac{6}{12}, \frac{2}{12}, \frac{3}{12}$.
29. $\frac{4}{28}, \frac{7}{28}$.
30. $\frac{3}{36}, \frac{4}{36}$.
33. $\frac{17066368}{2736407552}, \frac{11542664448}{2736407552}$, $\frac{222663595}{2736407552}$.
34. $\frac{2680}{7035}, \frac{15477}{7035}, \frac{4095}{7035}, \frac{4020}{7035}$.
35. $\frac{18900}{3780}, \frac{25200}{3780}, \frac{2240}{3780}$, $\frac{864}{3780}, \frac{2835}{3780}$.

(Art. **134**, p. 82.)

4. $\frac{7 \times \cancel{12} \times 2\cancel{0} \times 24}{\cancel{6}\cancel{0}\cancel{0} \quad 5\cancel{0}} = \frac{336}{5}$ gr., Ans.

5. $\frac{7 \times \cancel{12} \times 8 \times 3 \times 2\cancel{0}}{\cancel{6}\cancel{0}\cancel{0} \quad 5\cancel{0}} = \frac{336}{5}$ gr., Ans.

6. $\frac{1 \times \overset{3}{\cancel{24}} \times 6\cancel{0} \times 60}{\cancel{5}\cancel{6}\cancel{0} \quad 7} = \frac{1080}{7}$ sec., Ans.

7. $\frac{1 \times 4 \times 8 \times \cancel{2}}{\cancel{6} \quad 3} = \frac{32}{3}$ pt., Ans.

8. $\frac{1 \times 4 \times 2 \times 4}{23} = \frac{32}{23}$ gi., Ans.

9. $\frac{2 \times \overset{9}{\cancel{27}} \times 1728}{\cancel{21} \quad 7} = \frac{31104}{7}$ c. in., Ans.

10. $\frac{1 \times \overset{6}{\cancel{3}\cancel{0}} \times \overset{12}{\cancel{6}\cancel{0}} \times \overset{20}{\cancel{6}\cancel{0}}}{\cancel{525} \quad \cancel{1}\cancel{0}\cancel{5} \quad \cancel{21} \quad 7} = \frac{1440}{7}$ sec., Ans.

11. $\frac{3 \times 640 \times \cancel{4} \times 40 \times 121 \times 9 \times 144}{73 \quad \times \quad \cancel{4}} =$
$\frac{12043468800}{73}$ sq. in., Ans.

12. $\frac{19 \times 8 \times 10 \times 4 \times 25}{31} = \frac{152000}{31}$ li., Ans.

13. $\dfrac{1 \times \cancel{7} \times \cancel{24} \times \cancel{60} \times \cancel{60}}{\cancel{1814400}\ \ \cancel{2592}\ \ \cancel{108}\ \ \cancel{18}\ \ 3} = \dfrac{1}{3}$sec., Ans.

14. $\dfrac{4 \times \cancel{4} \times \overset{8}{\cancel{40}} \times 121}{\cancel{225}\ \ 45\ \ \times\ \ \cancel{4}} = \dfrac{3872}{45}$ sq. yd., Ans.

15. $\dfrac{\cancel{5} \times 4 \times \cancel{4} \times \cancel{9}}{\cancel{225}\ \ \cancel{45}\ \ 5 \times \cancel{4}} = \dfrac{4}{5}$in., Ans.

16. $\dfrac{1 \times \overset{4}{\cancel{12}} \times 30 \times 60 \times 60}{\cancel{21}\ \ 7} = \dfrac{432000}{7}$sec., Ans.

(ART. **135**, p. 83.)

4. $\dfrac{\cancel{336}\ \ \cancel{28}\ \ 7}{5 \times \underset{6}{\cancel{24}} \times 20 \times \cancel{12}} = \dfrac{7}{600}$lb., Ans.

5. $\dfrac{\cancel{336}\ \ \cancel{28}\ \ 7}{5 \times 20 \times 3 \times \underset{2}{\cancel{8}} \times \cancel{12}} = \dfrac{7}{600}$lb., Ans.

6. $\dfrac{\cancel{1080}\ \ \cancel{18}\ \ \cancel{3}}{7 \times \cancel{60} \times \underset{20}{\cancel{60}} \times \underset{4}{\cancel{24}}} = \dfrac{1}{560}$d., Ans.

7. $\dfrac{\cancel{32}\ \ \cancel{4}}{3 \times 2 \times \cancel{8} \times \cancel{4}} = \dfrac{1}{6}$bush., Ans.

8. $\dfrac{\cancel{32}\ \ \cancel{8}\ \ \cancel{4}}{23 \times \cancel{4} \times \cancel{2} \times \cancel{4}} = \dfrac{1}{23}$gal., Ans.

9. $\dfrac{\cancel{31104}\ \ \cancel{18}\ \ 2}{7 \times \cancel{1728} \times \cancel{27}\ \ 3} = \dfrac{2}{21}$c. yd., Ans.

10. $\dfrac{\cancel{1440}\ \ \cancel{24}\ \ \cancel{2}\ \ 1}{7 \times \cancel{60} \times \underset{5}{\cancel{60}} \times \underset{15}{\cancel{30}}} = \dfrac{1}{525}$s., Ans.

11. $\frac{12043468800 \times 4}{73 \times 144 \times 9 \times 121 \times 40 \times 4 \times 640} = \frac{3}{73}$sq. m., Ans.

12. $\frac{152000}{31 \times 25 \times 4 \times 10 \times 8} = \frac{19}{31}$m., Ans.

13. $\frac{1}{3 \times 60 \times 60 \times 24 \times 7} = \frac{1}{1814400}$wk., Ans.

14. $\frac{3872 \times 4}{45 \times 121 \times 40 \times 4} = \frac{4}{225}$a., Ans.

15. $\frac{4 \times 4}{5 \times 9 \times 4 \times 4} = \frac{1}{45}$yd., Ans.

16. $\frac{432000}{7 \times 60 \times 60 \times 30 \times 12} = \frac{1}{21}$circ., Ans.

(Art. 136, p. 84.)

3. $\frac{8}{15}$a. (= $\frac{8}{15}$r. × 4) = $\frac{32}{15}$r. = $2\frac{2}{15}$r.; $\frac{2}{15}$r. (= $\frac{2}{15}$rd. × 40) = $\frac{16}{3}$rd. = $5\frac{1}{3}$rd.; $\frac{1}{3}$rd. (= $\frac{1}{3}$yd. × $30\frac{1}{4}$) = $\frac{121}{12}$yd. = $10\frac{1}{12}$yd.; $\frac{1}{12}$yd. (= $\frac{1}{12}$ft. × 9) = $\frac{3}{4}$ft.; $\frac{3}{4}$ft. (= $\frac{3}{4}$in. × 144) = 108in.; ∴ $\frac{8}{15}$a. = 2r. 5rd. 10yd. 108in., Ans.

4. $\frac{9}{25}$lb. (= $\frac{9}{25}$oz. × 12) = $\frac{108}{25}$oz. = $4\frac{8}{25}$oz.; $\frac{8}{25}$oz. (= $\frac{8}{25}$dwt. × 20) = $\frac{32}{5}$dwt. = $6\frac{2}{5}$dwt.; $\frac{2}{5}$dwt. (= $\frac{2}{5}$gr. × 24) = $\frac{48}{5}$gr. = $9\frac{3}{5}$gr.; ∴ $\frac{9}{25}$lb. = 4oz. 6dwt. $9\frac{3}{5}$gr., Ans.

Note. — To abbreviate the operations in the remaining examples in this Article, *the multipliers will be carried in the mind*, only the results being written.

5. $\frac{7}{96}$lb. = $\frac{7}{8}$oz. = 7dr., Ans.

6. $\frac{73}{512}$lb. = $\frac{73}{32}$oz. = $2\frac{9}{32}$oz.; $\frac{9}{32}$oz. = $\frac{9}{2}$dr. = $4\frac{1}{2}$dr.; ∴ $\frac{73}{512}$lb. = 2oz. $4\frac{1}{2}$dr., Ans.

7. $\frac{3}{17}$m. = $\frac{24}{17}$fur. = $1\frac{7}{17}$fur.; $\frac{7}{17}$fur. = $\frac{70}{17}$ch. = $4\frac{2}{17}$ch.; $\frac{2}{17}$ch. = $\frac{8}{17}$rd. = $\frac{200}{17}$li. = $11\frac{13}{17}$li.; $\frac{13}{17}$li. = $\frac{10296}{1700}$in. = $6\frac{24}{425}$in.; ∴ $\frac{3}{17}$m. = 1fur. 4ch. 11 li. $6\frac{24}{425}$in., Ans.

8. $\frac{19}{20}$c. = $\frac{38}{5}$c. ft. = $7\frac{3}{5}$c. ft.; $\frac{3}{5}$c. ft. = $\frac{48}{5}$cu. ft. = $9\frac{3}{5}$cu. ft.; $\frac{3}{5}$cu. ft. = $\frac{5184}{5}$c. in. = $1036\frac{4}{5}$c. in.; ∴ $\frac{19}{20}$c. = 7c. ft. 9cu. ft. $1036\frac{4}{5}$c. in., Ans.

9. $\frac{6}{19}$yd. $=\frac{24}{19}$qr. $=1\frac{5}{19}$qr.; $\frac{5}{19}$qr. $=\frac{20}{19}$na. $=1\frac{1}{19}$na.; $\frac{1}{19}$na. $=\frac{9}{76}$in.; $\therefore$ $\frac{6}{19}$yd. $=$ 1qr. 1na. $\frac{9}{76}$in., Ans.

10. $\frac{53}{97}$gal. $=\frac{212}{97}$qt. $=2\frac{18}{97}$qt.; $\frac{18}{97}$qt. $=\frac{36}{97}$pt. $=\frac{144}{97}$gi. $=1\frac{47}{97}$gi.; $\therefore$ $\frac{53}{97}$gal. $=$ 2 qt. $1\frac{47}{97}$gi., Ans.

11. $\frac{21}{38}$bush. $=\frac{42}{19}$pk. $=2\frac{4}{19}$pk.; $\frac{4}{19}$pk. $=\frac{32}{19}$qt. $=1\frac{13}{19}$qt.; $\frac{13}{19}$qt. $=\frac{26}{19}$pt. $=1\frac{7}{19}$pt.; $\therefore$ $\frac{21}{38}$bush. $=$ 2pk. 1qt. $1\frac{7}{19}$pt., Ans.

12. $\frac{1}{7}$J.yr. $=\frac{1461}{784}$l. m. $=1\frac{677}{784}$l. m.; $\frac{677}{784}$l.m. $=\frac{677}{196}$wk. $=3\frac{89}{196}$wk.; $\frac{89}{196}$wk. $=\frac{89}{28}$d. $=3\frac{5}{28}$d.; $\frac{5}{28}$d. $=\frac{30}{7}$h. $=4\frac{2}{7}$h.; $\frac{2}{7}$h. $=\frac{120}{7}$m. $=17\frac{1}{7}$m.; $\frac{1}{7}$m. $=\frac{60}{7}$sec. $=8\frac{4}{7}$sec.; $\therefore$ $\frac{1}{7}$J. yr. $=$ 1 l. m. 3wk. 3d. 4h. 17m. $8\frac{4}{7}$sec., Ans.

13. $\frac{1}{9}$circ. $=\frac{4}{3}$s. $=1\frac{1}{3}$s.; $\frac{1}{3}$s. $=10°$; $\therefore$ $\frac{1}{9}$cir. $=$ 1s. 10°, Ans.

14. $\frac{3}{7}$l. $=\frac{9}{7}$m. $=1\frac{2}{7}$m.; $\frac{2}{7}$m. $=\frac{16}{7}$fur. $=2\frac{2}{7}$fur.; $\frac{2}{7}$fur. $=\frac{80}{7}$rd. $=11\frac{3}{7}$rd.; $\frac{3}{7}$rd. $=\frac{33}{14}$yd. $=2\frac{5}{14}$yd.; $\frac{5}{14}$yd. $=\frac{15}{14}$ft. $=1\frac{1}{14}$ft.; $\frac{1}{14}$ft. $=\frac{6}{7}$in. $=\frac{18}{7}$b.c. $=2\frac{4}{7}$b.c.; $\therefore$ $\frac{3}{7}$l. $=$ 1m. 2fur. 11rd. 2yd. 1ft. $2\frac{4}{7}$b.c., Ans.

15. $\frac{5}{24}$t. $=\frac{25}{3}$cu. ft. $=8\frac{1}{3}$cu. ft.; $\frac{1}{3}$cu. ft. $=$ 576c. in.; $\therefore$ $\frac{5}{24}$t. $=$ 8cu. ft. 576c in., Ans.

16. $\frac{19}{27}$c. yr. $=\frac{6935}{27}$d. $=256\frac{23}{27}$d.; $\frac{23}{27}$d. $=\frac{184}{9}$h. $=20\frac{4}{9}$h.; $\frac{4}{9}$h. $=\frac{80}{3}$m. $=26\frac{2}{3}$m.; $\frac{2}{3}$m. $=$ 40sec.; $\therefore$ $\frac{19}{27}$c. yr. $=$ 256d. 20h. 26m. 40sec., Ans.

17. $\frac{2879}{2880}$lb. $=\frac{2879}{240}$oz. $=11\frac{239}{240}$oz.; $\frac{239}{240}$oz. $=\frac{239}{30}$dr. $=7\frac{29}{30}$dr.; $\frac{29}{30}$dr. $=\frac{29}{10}$sc. $=2\frac{9}{10}$sc.; $\frac{9}{10}$sc. $=$ 18gr.; $\therefore$ $\frac{2879}{2880}$lb. $=$ 11oz. 7dr. 2sc. 18gr., Ans.

18. $\frac{47791}{64800}$circ. $=\frac{47791}{5400}$s. $=8\frac{4591}{5400}$s.; $\frac{4591}{5400}$s. $=\frac{4591}{180}°=25\frac{91}{180}°$; $\frac{91}{180}°=\frac{91}{3}'=30\frac{1}{3}'$; $\frac{1}{3}'=20''$; $\therefore$ $\frac{47791}{64800}$circ. $=$ 8s. 25° 30′ 20″, Ans.

(Art. **137**, p. 86.)

3. 108in. (= 108ft. ÷ 144) $=\frac{3}{4}$ft. (= $\frac{3}{4}$yd. ÷ 9) $=\frac{1}{12}$yd.; $10\frac{1}{12}$yd. (= $10\frac{1}{12}$rd. ÷ $30\frac{1}{4}$) $=\frac{1}{3}$rd.; $5\frac{1}{3}$rd. (= $5\frac{1}{3}$r. ÷ 40) $=\frac{2}{15}$r.; $2\frac{2}{15}$r. (= $2\frac{2}{15}$a. ÷ 4) $=\frac{8}{15}$a., Ans.

Note. — Omit divisors, setting down results.

4. $9\frac{3}{5}$gr. $=\frac{2}{5}$dwt.; $6\frac{2}{5}$dwt. $=\frac{8}{25}$oz., $4\frac{8}{25}$oz. $=\frac{9}{25}$lb., Ans.

5. 7dr. $= \frac{7}{8}$oz. $= \frac{7}{96}$lb., Ans.

6. $4\frac{1}{2}$dr. $= \frac{9}{32}$oz ; $2\frac{9}{32}$oz. $= \frac{73}{512}$lb., Ans.

7. $6\frac{24}{425}$in. $= \frac{13}{17}$li.; $11\frac{13}{17}$li. $= \frac{2}{17}$ch.; $4\frac{2}{17}$ch. $= \frac{7}{17}$fur; $1\frac{7}{17}$fur. $= \frac{3}{17}$m., Ans.

8. $1036\frac{4}{5}$c. in. $= \frac{3}{5}$cu. ft.; $9\frac{3}{5}$cu. ft. $= \frac{3}{5}$c. ft.; $7\frac{3}{5}$c. ft. $= \frac{19}{20}$c., Ans.

9. $\frac{9}{76}$in. $= \frac{1}{19}$na.; $1\frac{1}{19}$na. $= \frac{5}{19}$qr.; $1\frac{5}{19}$qr. $= \frac{6}{19}$yd., Ans.

10. $1\frac{47}{97}$gi. $= \frac{36}{97}$pt. $= \frac{18}{97}$qt.; $2\frac{18}{97}$qt. $= \frac{53}{97}$gal., Ans.

11. $1\frac{7}{19}$pt. $= \frac{13}{19}$qt.; $1\frac{13}{19}$qt. $= \frac{4}{19}$pk.; $2\frac{4}{19}$pk. $= \frac{21}{38}$bush., Ans.

12. $8\frac{4}{7}$sec. $= \frac{1}{7}$m.; $17\frac{1}{7}$m. $= \frac{2}{7}$h.; $4\frac{2}{7}$h. $= \frac{5}{28}$d.; $3\frac{5}{28}$d. $= \frac{89}{196}$wk.; $3\frac{89}{196}$wk. $= \frac{677}{784}$l. m.; $1\frac{677}{784}$l. m. $= \frac{1}{7}$J.yr., Ans.

13. $10° = \frac{1}{3}$s.; $1\frac{1}{3}$s. $= \frac{1}{9}$circ., Ans.

14. $2\frac{4}{7}$b.c. $= \frac{6}{7}$in. $= \frac{1}{14}$ft.; $1\frac{1}{14}$ft. $= \frac{5}{14}$yd.; $2\frac{5}{14}$yd. $= \frac{3}{7}$rd.; $11\frac{3}{7}$rd. $= \frac{2}{7}$fur.; $2\frac{2}{7}$fur. $= \frac{2}{7}$m.; $1\frac{2}{7}$m. $= \frac{3}{7}$l., Ans.

15. 576c. in. $= \frac{1}{3}$cu. ft.; $8\frac{1}{3}$cu. ft. $= \frac{5}{24}$t., Ans.

16. 40sec. $= \frac{2}{3}$m.; $26\frac{2}{3}$m. $= \frac{4}{9}$h.; $20\frac{4}{9}$h. $= \frac{23}{27}$d.; $256\frac{23}{27}$d. $= \frac{19}{27}$c. yr., Ans.

17. 18gr. $= \frac{9}{10}$sc.; $2\frac{9}{10}$sc. $= \frac{29}{30}$dr.; $7\frac{29}{30}$dr. $= \frac{239}{240}$oz.; $11\frac{239}{240}$oz. $= \frac{2879}{2880}$lb., Ans. by Rule 2.

$11 \times 8 + 7 = 95$; $95 \times 3 + 2 = 287$;

$$\frac{287 \times 20 + 18 = 5758}{12 \times 8 \times 3 \times 20 = 5760} = \frac{2879}{2880}\text{lb., Ans. by Rule 1.}$$

18. $20'' = \frac{1}{3}'$; $30\frac{1}{3}' = \frac{91}{180}°$; $25\frac{91}{180}° = \frac{4591}{5400}$s.; $8\frac{4591}{5400}$s. $= \frac{47791}{64800}$circ., Ans.

(Art. 138, p. 87.)

6.	$1\frac{3}{5}$	9.	$1\frac{12}{25}$	15.	$2\frac{100}{117}$	20.	$4\frac{4}{7}$
7.	$1\frac{1}{4}$	10.	$3\frac{25}{36}$	16.	$3\frac{251}{792}$	21.	$4\frac{139}{504}$
8.	$2\frac{9}{28}$	14.	$1\frac{587}{1980}$	17.	$2\frac{343}{1188}$		

24. $\frac{1}{7}$t. $= \frac{20}{7}$cwt.; $\frac{20}{7}$cwt. $+ \frac{1}{5}$cwt. $= \frac{107}{35}$cwt. $= 3\frac{2}{35}$cwt., Ans.

25. $\frac{2}{3}$bush. $= \frac{8}{3}$pk.; $\frac{2}{7}$qt. $= \frac{1}{28}$pk.; $\frac{8}{3} + \frac{3}{5} + \frac{1}{28} = 3\frac{127}{420}$pk., Ans.

26. $\frac{2}{5}$d. $= \frac{2}{35}$wk.; $\frac{1}{7}$h. $= \frac{1}{1176}$wk.; $\frac{3}{7}$m. $= \frac{1}{23520}$wk.; $\frac{1}{5}$wk. $+ \frac{2}{35}$wk. $+ \frac{1}{1176}$wk. $+ \frac{1}{23520}$wk. $= \frac{289}{1120}$wk. $= \frac{289}{160}$d. $\frac{867}{20}$h. $= \frac{2601}{1}$m., Ans.

27. $\frac{1}{7}$oz. $= \frac{1}{84}$lb.; $\frac{2}{7}$dr. $= \frac{1}{336}$lb.; $\frac{3}{8}$sc. $= \frac{1}{768}$lb.; $\frac{2}{5}$lb. $+ \frac{1}{84}$lb. $+ \frac{1}{336}$lb. $+ \frac{1}{768}$lb. $= \frac{11187}{26880}$lb. $= \frac{11187}{2240}$oz. $= \frac{11187}{280}$dr. $= \frac{33561}{280}$sc. $= \frac{33561}{14}$gr., Ans.

30.	$\frac{28}{55}$	32.	$4\frac{1}{30}$	35.	$27\frac{95}{264}$	37.	$26\frac{59}{72}$
31.	$1\frac{45}{88}$	33.	$4\frac{25}{36}$	36.	$19\frac{29}{72}$	38.	$47\frac{59}{210}$,

(Art. **139**, p. 90.)

3.	$\frac{15}{25} = \frac{3}{5}$	6.	$\frac{10}{57}$	11.	$\frac{47}{65}$	14.	$\frac{1028}{6715}$
4.	$\frac{58}{97}$	7.	$\frac{10}{27}$	12.	$\frac{25}{58}$	17.	$\frac{35}{99}$
5.	$\frac{7}{49} = \frac{1}{7}$	10.	$\frac{211}{2336}$	13.	$\frac{77}{928}$	19.	$\frac{11075}{43407}$

21. $\frac{1}{5}$qt. $= \frac{1}{20}$gal.; $\frac{1}{3}$gal. $- \frac{1}{20}$gal. $= \frac{17}{60}$gal. $= \frac{17}{15}$qt., Ans.

22. $\frac{1}{5}$cwt. $= \frac{1}{100}$t.; $\frac{1}{7}$t. $- \frac{1}{100}$t. $= \frac{93}{700}$t. $= \frac{93}{35}$cwt., Ans.

23. $\frac{1}{7}$h. $= \frac{1}{1176}$wk.; $\frac{1}{7}$wk. $- \frac{1}{1176}$wk. $= \frac{1169}{8232}$wk. $= \frac{1169}{1176}$d. $= \frac{167}{7}$h., etc., Ans.

24. $\frac{3}{7}$m. $= \frac{1}{3360}$d.; $\frac{2}{5}$d. $- \frac{1}{3360}$d. $= \frac{1343}{3360}$d. $= \frac{1343}{140}$h. $= \frac{4029}{7}$m., etc., Ans.

25. $\frac{2}{5}$lb. $= \frac{24}{5}$oz.; $\frac{3}{8}$sc. $= \frac{1}{64}$oz.; $\frac{24}{5}$oz. $- \frac{1}{64}$oz. $= \frac{1531}{320}$oz. $= \frac{1531}{40}$dr., etc., Ans.

28.	$\frac{4}{55}$	31.	$\frac{2806}{9875}$	37.	$14\frac{2}{15}$	39.	$203\frac{2}{5}$
29.	$\frac{21}{88}$	32.	$\frac{420}{6059}$	38.	$121\frac{119}{144}$	40.	$11\frac{4}{9}$
30.	$\frac{11}{30}$	36.	$19\frac{31}{304}$				

MISCELLANEOUS EXAMPLES IN FRACTIONS.

(Art. **140**, p. 91.)

1. $$\frac{3\frac{1}{4}}{6\frac{1}{2}} \times 72\tfrac{1}{2} \div \left(\tfrac{2}{3} \times \tfrac{3}{5} \times 9\tfrac{3}{8}\right) = \frac{\cancel{13}}{\cancel{4}} \times \frac{\cancel{2}}{\cancel{13}} \times \frac{\overset{29}{\cancel{145}}}{\cancel{2}} \times \frac{\cancel{3}}{\cancel{2}}$$

$$\times \frac{\cancel{5}}{\underset{3}{\cancel{3}}} \times \frac{\cancel{8}}{\underset{\cancel{15}}{\cancel{75}}} = \frac{29}{3} = 9\tfrac{2}{3}, \text{ Ans.}$$

2. $\frac{8\frac{2}{7}}{4\frac{1}{7}} \div \frac{1}{2} = 2 \times 2 = 4;\ 4 \times \frac{1}{4} \times 8\frac{2}{3} = 8\frac{2}{3}$, Ans.

3. $\frac{3}{5}$£ = 12s.; $\frac{1}{3}$d. = $\frac{1}{36}$s.; $\frac{2}{5}$qr. = $\frac{1}{120}$s.; 12s. + $\frac{2}{7}$s. + $\frac{1}{36}$s. + $\frac{1}{120}$s. = 12 $\frac{811}{2520}$s., Ans.

4. $7\frac{1}{3}$in. = $\frac{11}{18}$ft.; $2\frac{1}{4}$ft. + $\frac{11}{18}$ft. = $2\frac{31}{36}$ft. = $\frac{103}{108}$yd.; $2\frac{1}{7}$yd. + $\frac{103}{108}$yd. = $3\frac{73}{756}$yd. = $\frac{2341}{4158}$rd.; $4\frac{1}{5}$rd. + $\frac{2341}{4158}$rd. = $4\frac{15863}{20790}$rd. = $\frac{99023}{831600}$fur., Ans.

5. $\frac{1}{2} \times \frac{9}{11} = \frac{9}{22};\ \frac{8\frac{2}{3}}{4\frac{1}{5}} \times 2\frac{1}{10} = \frac{26}{3} \times \frac{5}{21} \times \frac{21}{10} = \frac{13}{3};\ \frac{13}{3} - \frac{9}{22} = \frac{286 - 27}{66} = \frac{259}{66} = 3\frac{61}{66}$, Ans.

6. $\frac{2}{7}$gal. = $\frac{8}{7}$qt. = $1\frac{1}{7}$qt.; $\frac{1}{7}$qt. = $\frac{2}{7}$pt. = $\frac{8}{7}$gi. = $1\frac{1}{7}$gi.; ∴ $\frac{2}{7}$gal. = 1qt. $1\frac{1}{7}$gi., Ans.

7. $12\frac{1}{2}$c. × $12\frac{1}{2}$ = $156\frac{1}{4}$c. = \$1.56$\frac{1}{4}$, Ans.

8. $23\frac{1}{2}$c. × $33\frac{1}{2}$ = $787\frac{1}{4}$c. = \$7.87$\frac{1}{4}$, Ans.

9. \$$\frac{7}{4}$ × $\frac{8}{7}$ × $\frac{127}{5}$ = \$$\frac{254}{5}$ = \$$50\frac{4}{5}$ = \$50.80, Ans.

Explanation. — The cost, \$$\frac{7}{4}$, divided by the quantity, $\frac{7}{8}$, gives the price of 1 bushel, and this multipled by $25\frac{2}{5}$ gives the value of $25\frac{2}{5}$ bushels.

Note. — A skillful mathematician will first solve an analytic example *mentally, indicating the relations of the numbers to each other, and afterward* cancel, multiply, divide, etc.

10. \$$\frac{105}{16}$ × $\frac{4}{35}$ × $\frac{55}{2}$ = \$$\frac{165}{8}$ = \$$20\frac{5}{8}$ = \$20.62$\frac{1}{2}$, Ans.

11. Each boy receives $\frac{5}{5}$ as much as a boy; ∴ 7 boys receive 7 times $\frac{5}{5}$ = $\frac{35}{5}$ as much as a boy. Again, each man receives $\frac{7}{5}$ as much as a boy; ∴ 5 men receive 5 times $\frac{7}{5}$ = $\frac{35}{5}$ as much as a boy. Thus we see that *all* of the men receive just as much as *all* of the boys; ∴ first divide the \$144 into two equal parts, viz. \$72 and \$72, and then each man receives \$72 ÷ 5 = \$$14\frac{2}{5}$, and each boy \$72 ÷ 7 = \$$10\frac{2}{7}$, Ans.

12. $7\frac{1}{2}$ is $\frac{1}{2}$ of 15, ∴ $7\frac{1}{2}$bbl. will cost $\frac{1}{2}$ of \$157.50 = \$78.75, Ans.

13. $2\frac{1}{2}$c. × $7\frac{1}{2} = 18\frac{3}{4}$c., Ans.

14. 42c. ÷ $\frac{6}{11} =$ 77c.; $\$8\frac{17}{200} \div$ 77c. $= 10\frac{1}{2}$, No. bush., Ans.

15. If 32 is $\frac{8}{9}$, then 4 is $\frac{1}{9}$ and 36 is $\frac{9}{9}$; $\frac{1}{3}$ of 12 is 4, and 4 in 36, 9 times, Ans.

16. If 36 is $\frac{4}{7}$, then 9 is $\frac{1}{7}$ and 63 is $\frac{7}{7}$; 9 in 63, 7 times, Ans.

17. $\frac{1}{7}$ of 28 is 4 and $\frac{2}{7}$ are 8; if 8 is $\frac{4}{9}$, then 2 is $\frac{1}{9}$ and 18 is $\frac{9}{9}$; $\frac{1}{5}$ of 15 is 3, and 3 in 18, 6, Ans.

18. $\frac{1}{3}$ of 15 is 5 and $\frac{5}{3}$ are 25; if 25 is $\frac{5}{11}$, then 5 is $\frac{1}{11}$ and 55 is $\frac{11}{11}$; $\frac{1}{8}$ of 88 is 11, and 11 in 55, 5, Ans.

19. $\$\frac{5}{4} \times 11\frac{1}{2} = \$\frac{115}{8} = \$14\frac{3}{8} \doteq \$14.37\frac{1}{2}$, Ans.

20. If $\$87\frac{1}{2}$ is $\frac{7}{8}$, then $\frac{1}{8}$ is $\frac{1}{7}$ of $\$87\frac{1}{2} = \$12\frac{1}{2} = \$12.50$, Ans., for, if the watch was sold for $\frac{7}{8}$ of the cost, the remaining $\frac{1}{8}$ was lost.

21. $$8)\frac{2160 \mid \overset{6)}{270} \mid \overset{9)}{45} \mid 5}{4752 \mid 594 \mid 99 \mid 11}, \text{ Ans.}$$

22. $128 \times \frac{9}{11} = 104\frac{8}{11}$, 1st Ans.; $\$\frac{11}{2} \times \frac{9}{11} = \$\frac{9}{2} = \$4.50$, 2d Ans.

23. $(\$64\frac{19}{72} \div 73\frac{4}{9}) \times 64 = \$\frac{4627}{72} \times \frac{9}{661} \times 64 = \$56$, Ans.

24. $$£\frac{2}{3 \times 4 \times 12 \times 20} = £\frac{1}{1440}, \text{ Ans.}$$

25. $27\frac{3}{4}$c. × $72\frac{1}{5} = 2003\frac{11}{20}$c. $= \$20.03\frac{11}{20}$, Ans.

27. $1\frac{3}{4}$pt. × 12 = 21pt.; 63 gal. = 504pt.; 504 ÷ 21 = 24, Ans.

28. $\$96\frac{1}{8} \times 73\frac{1}{11} = \$7025\frac{19}{22} = \$7025.86\frac{4}{11}$, Ans.

29. $6\frac{1}{4}$c. × $1763\frac{1}{9} = 11019\frac{4}{9}$c. $= \$110.19\frac{4}{9}$, Ans.

30. $(19\frac{1}{4}\text{m.} \div 8\frac{7}{8}) \times 64\frac{11}{32} = \frac{77}{4}\text{m.} \times \frac{8}{71} \times \frac{2059}{32} = 139\frac{9}{16}\text{m.}$, Ans.

EXPLANATION. — First divide $19\frac{1}{4}$m. by $8\frac{7}{8}$, to see how far the given weight may be carried for $1, and then multiply by $64\frac{11}{32}$, to see how far it may be carried for $\$64\frac{11}{32}$.

31. Since the *exterior* dimensions are 2 inches greater than the *interior*, the top and bottom are respectively 3ft. 8in. = $3\frac{2}{3}$ft. in length and width; each *side* is $3\frac{2}{3}$ft. long and $2\frac{1}{4}$ft. wide; and each *end* $3\frac{1}{2}$ft. long and $2\frac{1}{4}$ft. wide. Hence,

$$\frac{11 \times 11 \times 2 \times 36}{3 \times 3} = \left\{\begin{array}{l}\text{968sq. ft., top and bottom of 36}\\ \text{boxes.}\end{array}\right.$$

$$\frac{11 \times 9 \times 2 \times 36}{3 \times 4} = \text{594sq. ft., 2 sides of 36 boxes.}$$

$$\frac{7 \times 9 \times 2 \times 36}{2 \times 4} = \text{567sq. ft., 2 ends of 36 boxes.}$$

2129sq. ft., Ans.

32. The top and bottom are respectively $3\frac{1}{2}$ft. in length and width; each *side* is $3\frac{1}{2}$ft. long and 2ft. 1in. = $2\frac{1}{12}$ft. wide; and each *end* 3ft. 4in. = $3\frac{1}{3}$ft. long and $2\frac{1}{12}$ft. wide. Hence,

$$\frac{7 \times 7 \times 2 \times 36}{2 \times 2} = \left\{\begin{array}{l}\text{882sq. ft., top and bottom of 36}\\ \text{boxes.}\end{array}\right.$$

$$\frac{7 \times 25 \times 2 \times 36}{2 \times 12} = \text{525sq. ft., 2 sides of 36 boxes.}$$

$$\frac{10 \times 25 \times 2 \times 36}{3 \times 12} = \text{500sq. ft., 2 ends of 36 boxes.}$$

1907sq. ft., 1st Ans.

The capacity of one box of each kind may be found by multiplying its three interior dimensions together, and then the *difference* between these products multiplied by 36 will give the 2d Ans. But, more easily, since one of the smaller boxes will exactly *fit into* one of the larger, it is manifest that one of the larger boxes will hold just what one of the smaller ones will hold, *plus* the *boards which compose the smaller box;* i. e. the difference in the capacities of the two sets of boxes is just the *volume of the boards used in making the smaller boxes.* Now 1ft. of board equals 144c.in. of wood, ∴ 144c. in. × 1907 = 274608c. in., 2d Ans.

33. $\$\frac{770 \times 4 \times 3 \times 5}{100 \times 11 \times 5 \times 6} = \$\frac{7}{5} = \$1\frac{2}{5} = \$1.40$, Ans.

34. $\frac{12\text{a.} \times 3 \times 2}{5 \times 3} = \frac{24\text{a.}}{5} = 4\frac{4}{5}\text{a.}$, Ans.

35. $1640\text{m.} \times 1\frac{43}{82} = 2500\text{m.}$, length of cable.

$$\frac{2500\text{m.} \times 21 \times 25 \times 7 \times 18}{20 \times 24} = 344531\tfrac{1}{4}\text{m.}, \left\{\begin{array}{l}\text{length of } \textit{iron} \\ \quad\text{wire.}\end{array}\right.$$

$$\begin{array}{rl} 2500\text{m.} \times 7 = \underline{\ 17500\text{ m.}\ }, & \text{length of } \textit{copper} \text{ wire.} \\ 362031\tfrac{1}{4}\text{m.}, & \text{Ans.} \end{array}$$

EXPLANATION. — Multiplying a number by $\frac{21}{20}$ is the same, in effect, as adding $\frac{1}{20}$ of the number to the number. So multiplying by $\frac{25}{24}$ adds $\frac{1}{24}$. The wires are first twisted together, and *afterward* the bundles of *iron* wire run spirally; hence it is not the 2500m. that is to be increased by $\frac{1}{24}$ of itself, but the $\frac{21}{20}$ of the 2500m.

36. 1t. = 32000oz.; 1m. = 5280ft.; 32000oz. ÷ 5280 = $6\frac{2}{33}$oz., Ans.

37. $3\frac{1}{4}$bush. × $7\frac{1}{5}$ = $23\frac{2}{5}$bush., Ans.

38. $\$1\frac{2}{5} \times 13\frac{1}{4} = \$18\frac{11}{20} = \$18.55$, Ans.

39. $12\frac{1}{2} \div 3\frac{1}{8} = 4$, i. e. the shadow is $\frac{1}{4}$ as long as the pole; $\therefore 133\frac{1}{8}\text{ft.} \div 4 = 33\frac{9}{32}\text{ft.}$, Ans.

40. The pole is 4 times as long as the shadow; ∴ the steeple is $33\frac{9}{32}\text{ft.} \times 4 = 133\frac{1}{8}\text{ft.}$ long, Ans.

41. 7yd. × $3\frac{1}{2}$ = $24\frac{1}{2}$yd. of 2d kind.

$\$24\frac{1}{2} \div 8 = \$3\frac{1}{16}$, price of 1st kind, 1st Ans.

$\$3\frac{1}{16} \times \frac{3}{5} = \$1\frac{67}{80}$, price of 2d kind, 2d Ans.

$$\begin{array}{rl} \$3\frac{1}{16} \times 7 \quad = \$21\frac{7}{16}, & \text{cost of the 7 yd.} \\ \$1\frac{67}{80} \times 24\frac{1}{2} = \underline{\$45\frac{3}{160}}, & \text{cost of the } 24\tfrac{1}{2}\text{yd.} \\ \$66\frac{73}{160}, & \text{total cost, 3d Ans.} \end{array}$$

42. $\frac{3}{7} \times \frac{1}{2} \times \frac{7}{9} = \frac{1}{6}$; $\frac{4}{5} \times \frac{7}{8} \times \frac{10}{14} = \frac{1}{2}$; ∴ $\frac{1}{6}$ of the value of the cargo, viz. \$12000, is $\frac{1}{2}$ of the worth of the ship; hence the ship is worth \$12000 × 2 = \$24000, Ans.

43. $\frac{3}{8}+\frac{2}{7}+\frac{1}{5}+\frac{1}{9}=\frac{945+720+504+280}{2520}=\frac{2449}{2520}$; $\frac{2520}{2520}-\frac{2449}{2520}=\frac{71}{2520}$. Now the 142 persons are $\frac{71}{2520}$ of the population; ∴ $\frac{1}{2520}$ of the population is $142 \div 71 = 2$, and $\frac{2520}{2520}$ of it $= 2520$ times $2 = 5040$, Ans.

44. $(16\frac{1}{2}\text{ft.} + 15\text{ft}) \times 2 = 63$ft., distance round the room.
$63 \times 8\frac{2}{3} = 546$, area in sq. ft. of the 4 walls.
$16\frac{1}{2} \times 15 = 247\frac{1}{2}$, area in sq. ft. of the floor or ceiling.
$793\frac{1}{2}$sq. ft., area of walls and ceiling.
$63 - 3\frac{1}{4} \times 4 = 50$ft., length of base boards.
$50 \times \frac{3}{5} = 30$sq. ft., area of base boards.
$4\frac{1}{3} \times 3 \times 4 = 52$sq. ft., area of 4 windows.
$8 \times 3\frac{1}{4} \times 4 = 104$sq. ft., area of 4 doors.
186sq. ft., area of base, windows, and doors.
$793\frac{1}{2} - 186 = 607\frac{1}{2}$sq. ft. $= 67\frac{1}{2}$sq. yd., to be plastered.
$247\frac{1}{2} + 104 + 30 = 381\frac{1}{2}$sq. ft. to be painted.
$15\frac{1}{3}$c. $\times 12 \times 4 =$ \$7.36, cost of glazing.
$3\frac{1}{4}$c. $\times 381\frac{1}{2} =$ \$$12.39\frac{7}{8}$, cost of painting.
$22\frac{1}{2}$c. $\times 67\frac{1}{2} =$ \$$15.18\frac{3}{4}$, cost of plastering.
\$$34.94\frac{5}{8}$, Ans.

45. $247\frac{1}{2} \div 9 = 27\frac{1}{2}$, No. yd.; \$$1.33\frac{1}{3} \times 27\frac{1}{2} =$ \$$36.66\frac{2}{3}$, Ans.

46. $546 - 186 = 360$sq. ft. to be papered; $17\frac{1}{7} \times 3 \times 1\frac{3}{4} = 90$sq. ft. in 1 roll of paper; $360 \div 90 = 4$, No. rolls; \$$\frac{9}{20}$ + \$$\frac{1}{6}$ = \$$\frac{37}{60}$, cost of 1 roll; \$$\frac{37}{60} \times 4 =$ \$$\frac{37}{15}$ = \$$2\frac{7}{15}$, Ans.

47. $48\frac{3}{4} + 28\frac{1}{2} + 25\frac{3}{16} + 56\frac{3}{8} = 158\frac{13}{16}$, No. lb., 1st Ans.
$22\frac{1}{2}$c. $\times 158\frac{13}{16} =$ \$$35.73\frac{9}{32}$, 2d Ans.

48. $6\frac{1}{4}$c $\times 22 =$ \$$1.37\frac{1}{2}$
\$$\frac{1}{3} \times 10 =$ \$$3.33\frac{1}{3}$
$4\frac{1}{2}$c. $\times 50 =$ \$2.25
\$$6\frac{1}{8}$ = \$$6.12\frac{1}{2}$
\$$3\frac{3}{4}$ = \$3.75
\$$16.83\frac{1}{3}$

\$20 − \$$16.83\frac{1}{3}$ = \$$3.16\frac{2}{3}$, cost of 38yd. sheeting; ∴ \$$3.16\frac{2}{3} \div 38 = 8\frac{1}{3}$c., Ans.

49. $5280\text{ft.} \times 17\frac{3}{4} = 93720\text{ft.}$ in $17\frac{3}{4}$ miles; $93720 \div 9\frac{1}{3} = 10041\frac{3}{7}$, Ans.

50. $95000000 \div 37\frac{1}{2} = 2533333\frac{1}{3}$, i. e. 2533333h. 20m.; $2533333 \div 24 = 105555\frac{13}{24}$, i. e. 105555d. 13h.; $105555 \div 365\frac{1}{4} = 288\frac{363}{365\frac{1}{4}}$, i. e. 288yr. 363d. Hence, the time is 288yr. 363d. 13h. 20m., Ans.

51. $142\frac{1}{4}\text{a.} - 53\frac{7}{8}\text{a.} = 88\frac{3}{8}\text{a.}$, Ans.

Income.

		bush		bush.	\$		\$
52.	Potatoes,	250	$\times 2\frac{1}{2} =$	625;	$\frac{5}{8}$	$\times 625 =$	$390.62\frac{1}{2}$
	Corn,	$56\frac{1}{4}$	$\times 3\frac{1}{4} =$	$182\frac{13}{16}$;	$\frac{7}{8}$	$\times 182\frac{13}{16} =$	$159.96\frac{3}{32}$
	Wheat,	$32\frac{1}{4}$	$\times 3\frac{1}{2} =$	$112\frac{7}{8}$;	$1\frac{5}{8}$	$\times 112\frac{7}{8} =$	$183.42\frac{3}{16}$
	Rye,	25	$\times 1\frac{3}{4} =$	$43\frac{3}{4}$;	1	$\times 43\frac{3}{4} =$	43.75
	Oats,	$62\frac{1}{2}$	$\times 1\frac{3}{4} =$	$109\frac{3}{8}$;	$\frac{9}{20}$	$\times 109\frac{3}{8} =$	$49.21\frac{7}{8}$
	Hay,	$2\frac{1}{4}$t.	$\times 20\frac{1}{2} =$	$46\frac{1}{8}$t.;	$12\frac{1}{2}$	$\times 46\frac{1}{8} =$	$576.56\frac{1}{4}$
	Sheep,				4c.	$\times 25 \times 56 =$	56.00
	Cows,				25c.	$\times 25 \times 12 =$	75.00
	Oxen,				40c.	$\times 25 \times 4 =$	40.00
	Horses,				50c.	$\times 25 \times 3 =$	37.50
				Total income,			$\$1612.03\frac{29}{32}$

Expenses.

Potatoes,	$\$35 \times 2\frac{1}{2} =$	\$ 87.50	
Corn,	$\$35 \times 3\frac{1}{4} =$	\$113.75	
Wheat,	$\$33 \times 3\frac{1}{2} =$	\$115.50	
Rye,	$\$25 \times 1\frac{3}{4} =$	\$ 43.75	
Oats,	$\$15 \times 1\frac{3}{4} =$	\$ 26.25	
Hay,	$\$6 \times 20\frac{1}{2} =$	\$123.00	
Taxes,		\$ 32.00	
	Total expense,		\$541.75
	Net profits,	Ans.,	$\$1070.28\frac{29}{32}$

53. $\frac{4}{6}$ of $\frac{3}{8} = \frac{1}{4}$; if $\frac{1}{4} = \$3000$, then $\frac{4}{4} = \$12000$, Ans.

54. $\frac{5}{6}$ of $\$48000 = \$40000$; if $\frac{10}{3} = \$40000$ then $\frac{1}{3} = 4000$ and $\frac{3}{3} = \$12000$, Ans.

55. $\frac{1}{2} + \frac{1}{4} + \frac{1}{5} = \frac{19}{20}$; $\therefore \frac{1}{20} = 10$ and $\frac{20}{20} = 200$, Ans.

56. $\frac{3}{4} + \frac{5}{7} + \frac{8}{9} - \frac{2}{3} + \frac{1}{5} + \frac{1}{2} = \frac{3007}{1260}$; $\frac{3007}{1260} \times 5\frac{1}{4} = 12\frac{127}{240}$, Ans.

EXPLANATION. — The sign $\underset{.}{-}$ affects only the $\frac{2}{3}$; ∴ first take the $\frac{2}{3}$ from $\frac{8}{9}$, leaving $\frac{2}{9}$, and then add $\frac{3}{4}$, $\frac{5}{7}$, $\frac{2}{9}$, $\frac{1}{5}$, and $\frac{1}{2}$, giving $\frac{3007}{1260}$.

57. $\frac{3}{4} + \frac{5}{7} = \frac{41}{28}$; $\frac{2}{9} + \frac{1}{5} + \frac{1}{2} = \frac{83}{90}$; $\frac{83}{90} \times 5\frac{1}{4} = \frac{581}{120}$; $\frac{41}{28} + \frac{581}{120} = \frac{5297}{840} = 6\frac{257}{840}$, Ans.

58. $\frac{8}{13} \div 2 = \frac{4}{13}$; $\frac{5}{12} \times 4 = \frac{5}{3}$; $\frac{1}{9} \div \frac{1}{3} = \frac{1}{3}$; $\frac{2}{5} \times \frac{3}{2} = \frac{3}{5}$; $\frac{4}{13} + \frac{5}{3} = \frac{77}{39}$; $\frac{1}{3} + \frac{3}{5} = \frac{14}{15}$; $\frac{77}{39} - \frac{14}{15} = \frac{203}{195} = 1\frac{8}{195}$, Ans.

EXPLANATION. — In this example there are *four independent expressions.* Each of these expressions is reduced to a simple form; then the first two, being *positive* (i. e. having the *plus sign*), are added together, and the last two, being *negative* (i. e. having the *minus sign*), are added together, and, finally the sum of the negatives is taken from the sum of the positives.

59. $\frac{4}{7} \div \frac{5}{2} = \frac{8}{35}$; $\frac{8}{7} \div 12 = \frac{2}{21}$; $\frac{8}{35} - \frac{2}{21} = \frac{14}{105} = \frac{2}{15}$, Ans.

60. $\frac{3}{4}$ of $\frac{7\frac{1}{8}}{14\frac{1}{4}} = \frac{3}{4}$ of $\frac{1}{2} = \frac{3}{8}$; $\frac{5\frac{4}{7}}{7\frac{4}{5}} \div 1\frac{3}{7} = \frac{39}{7} \times \frac{5}{39} \times \frac{7}{10} = \frac{1}{2}$; $\left[\frac{1}{5} \text{ of } \frac{5}{7} + \frac{1}{2} \text{ of } \frac{3}{2}\right] \div \frac{5}{8} = \left[\frac{1}{7} + \frac{3}{4}\right] \div \frac{5}{8} = \frac{25}{28} \times \frac{8}{5} = \frac{10}{7}$; $\frac{\frac{3}{8}}{3} \times 8 = \frac{1}{8} \times 8 = 1$; taking 1 from $\frac{10}{7}$ leaves $\frac{3}{7}$; ∴ $\frac{3}{8} + \frac{1}{2} + \frac{3}{7} = \frac{73}{56} = 1\frac{17}{56}$, Ans.

EXPLANATION. — Here are four independent expressions. The first is reduced to $\frac{3}{8}$, the second to $\frac{1}{2}$, the third to $\frac{10}{7}$, and the fourth to -1. The -1 is first united to the $\frac{10}{7}$, giving $\frac{3}{7}$, and then $\frac{3}{8}$, $\frac{1}{2}$, and $\frac{3}{7}$ are added.

61. $\frac{4}{7}$s. $= \frac{1}{35}$£; $\frac{1}{5}$d. $= \frac{1}{1200}$£; $\frac{1}{4}$qr. $= \frac{1}{3840}$£; $\frac{3}{8}$£ $+ \frac{1}{35}$£ $+ \frac{1}{1200}$£ $- \frac{1}{3840}$£ $= \frac{54317}{134400}$£ $= \frac{54317}{6720}$s. $= \frac{54317}{560}$d. $= \frac{54317}{140}$qr., Ans.

62. $27\frac{3}{4} \times 5\frac{1}{3} = 148$, Ans.

63. $87\frac{1}{5} - 18\frac{3}{4} = 68\frac{9}{20}$, Ans.

64. $17\frac{2}{3} + 18\frac{1}{4} = 35\frac{11}{12}$, Ans.

65. $48\frac{4}{5} \div 24\frac{2}{5} = 2$, Ans.

66. $47\frac{3}{5} \div 12\frac{1}{3} = 3\frac{159}{185}$, Ans.

67. $17\frac{2}{3} \div 15\frac{3}{5} = 1\frac{31}{234}$, Ans.

68. $\frac{129}{4} \times \frac{106}{7} \times \frac{99}{5} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{6} = \frac{225621}{70} = 3223\frac{11}{70}$, Ans.

69. $\frac{2}{3}$ of $\$\frac{39}{5} = \$\frac{26}{5} = \$5\frac{1}{5} = \$5.20$, Ans.

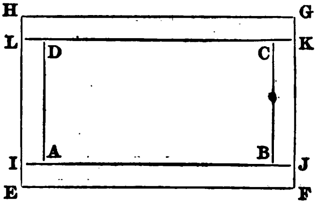
70. $8\frac{1}{4} \times \frac{11}{12} \div \frac{3}{5} = \frac{33}{4} \times \frac{11}{12} \times \frac{5}{3} = \frac{605}{48} = 12\frac{29}{48}$yd., Ans.

71. $24\frac{1}{2} \times 12\frac{1}{2} \times 1\frac{1}{2} \div 9\frac{4}{5} = \frac{49}{2} \times \frac{25}{2} \times \frac{3}{2} \times \frac{5}{49} = \frac{375}{8} = 46\frac{7}{8}$d., Ans.

72. $1\frac{3}{5}$ times 10rd. = 16rd., length of garden.

(10rd. + 16rd.) × 2 = 52rd. = 858ft., distance round the garden.

A B C D represents the *garden*, and since the wall is *outside* of the boundary line of the garden, each *side* wall, E F J I and L K G H, is 2 times $3\frac{1}{2}$ft. = 7ft. *longer* than the garden; ∴ add twice 7ft. = 14ft. to the length of the boundary line, and we have 858ft. + 14ft. = 872ft. for the length of the wall.



$$\text{Hence, } \frac{872 \times 7 \times 11 \times 3\text{c.}}{2 \times 4 \times 4} = \frac{25179\text{c.}}{4} = \$62.94\tfrac{3}{4}, \text{ Ans.}$$

73. In Ex. 73, E F G H, instead of A B C D represents the garden, and, since the ditch is *within* the boundary line, each *end* ditch, I A D L and B J K C, is 7ft. *shorter* than the width of the garden; ∴ subtract 14ft. from the distance round the garden, and we have 858ft. — 14ft. = 844ft. for the length of the ditch. Hence,

$$\frac{844 \times 7 \times 11 \times 3\text{c.}}{2 \times 4 \times 8} = \frac{48741\text{c.}}{16} = \$30.46\tfrac{5}{16}, \text{ Ans.}$$

74. A can do $\frac{1}{60}$ of the work in 1 hour, or $\frac{11}{60}$ in 1 day of 11 hours; B can do $\frac{1}{8}$ of it in 1 day of 11 hours; ∴ A and B can do $\frac{11}{60} + \frac{1}{8} = \frac{37}{120}$ of it in 1 day. Hence, $\frac{120}{120} \div \frac{37}{120} = \frac{120}{37} = 3\frac{9}{37}$, No. of days, Ans.

75. $\$3\frac{1}{4} \times 12\frac{1}{2} = \$40.62\frac{1}{2}$, cost of cloth. $\$62.50 - \$40.62\frac{1}{2} = \$21.87\frac{1}{2}$, cost of sugar. $\$21.87\frac{1}{2} \div 12\frac{1}{2}$c. $= 175$, No. lb. of sugar, Ans.

76. The horse cost $\frac{9}{8}$ of his own price; ∴ the horse and oxen cost $\frac{17}{8}$ of the price of the horse; i.e. \$340 is $\frac{17}{8}$ of the price of the horse, and $\$340 \div 17 = \$20$ is $\frac{1}{8}$ of the price of the horse. Hence, $\$20 \times 8 = \$160$, price of horse; $\$20 \times 9 = \$180$, price of oxen; $\$20 \times 3 = \$60$, price of wagon, Ans.

77. $7\frac{3}{8} \times 7\frac{1}{4} = \frac{1711}{32}$; $3\frac{1}{2} + 3\frac{1}{2} \times 3\frac{1}{2} = \frac{7}{2} + \frac{49}{4} = \frac{63}{4}$; $\frac{1711}{32} - \frac{63}{4} = \frac{1207}{32} = 37\frac{23}{32}$sq. rd., Ans.

78. $\frac{13}{13} - \frac{8}{13} = \frac{5}{13}$, B's share; $\frac{8}{13} - \frac{5}{13} = \frac{3}{13} =$ 7a. 3r. $15\frac{1}{2}$rd. $= 7\frac{271}{320}$a.; ∴ $\frac{1}{13} = 7\frac{271}{320}$a. $\div 3 = 2\frac{197}{320}$a., and B's share, $\frac{5}{13} = 2\frac{197}{320}$a. $\times 5 = 13\frac{5}{64}$a. $=$ 13a. $12\frac{1}{2}$rd., Ans.

79. $\frac{5}{5} - \frac{1}{5} = \frac{4}{5}$, 1st remainder; $\frac{1}{8}$ of $\frac{4}{5} = \frac{1}{10}$, No. given to 2d companion; $\frac{4}{5} - \frac{1}{10} = \frac{7}{10}$, 2d remainder; $\frac{1}{4}$ of $\frac{7}{10} = \frac{7}{40}$, No. lost; $2\frac{1}{7}$ times $\frac{7}{40} = \frac{15}{40}$, No. sold; ∴ No. disposed of, $\frac{1}{5} + \frac{1}{10} + \frac{7}{40} + \frac{15}{40} = \frac{34}{40}$, and the 6 marbles must be the remaining $\frac{6}{40}$; ∴ 1 marble is $\frac{1}{40}$, and $\frac{40}{40} =$ 40 marbles, Ans.

80. $5 + \frac{3}{5}$ of $5 = 8$, final No. in family;

$$\frac{3 \times 5 \times 195 \times 8}{5 \times 22 \times 8 \times 5} = \frac{117}{22} = 5\tfrac{7}{22}\text{bbl., Ans.}$$

81. $$\frac{13 \times 13 \times 8 \times 1024}{5 \times 8 \times 7} = \frac{173056}{35} = 4944\tfrac{16}{35}\text{yd., Ans.}$$

82. $9\frac{3}{7} + 7\frac{1}{2} - \frac{2}{3}$ of $\frac{3}{5}$ of $10\frac{5}{7} = \frac{237}{14} - \frac{30}{7} = \frac{177}{14} = 12\frac{9}{14}$, Ans.

DECIMAL FRACTIONS.

(ART. **152**, p. 100.)

2.	.013	7.	3000.003
3.	.0000089	8.	1250.6
4.	.252	9.	.777
5.	.0663	10.	8000.000018.
6.	500.005		

(ART. **153**, p. 100.)

1. Eight hundred and sixty-five and four ten-thousandths.
2. Forty-two and four thousand, two hundred and forty-seven ten-thousandths.
3. Five hundred and decimal five ten-thousandths.
4. Seven hundred and ninety-six and six thousand, seven hundred and four ten-thousandths.
5. Four and six ten-millionths.
6. Eighty-seven thousand, six hundred and fifty-four and two hundred-thousanths.
7. Forty thousand and decimal four millionths.
8. Two hundred and seventy-eight and forty-six million, nine hundred and forty-three thousand, eight hundred and twenty-seven hundred millionths.
9. Two hundred and two and four tenths.
10. Ninety-nine and nine hundred ninety-nine thousand, nine hundred and ninety-nine millionths.

(ART. **154**, p. 101.)

4.	2072103.360104	9.	2.8568432723
5.	571711.89162714434	10.	5371.721
6.	1637.25074	11.	35000213.358004
7.	4922.06814	12.	153971.17741600005
8.	104452.704006		

(Art. **155**, p. 102.)

4.	832.670128	8.	.87218446
5.	869510.12305713	9.	874368.465731
6.	53.854	10.	1.5200642
7.	65999999.999934		

(Art. **156**, p. 103.)

9.	180.3774	16.	34672800000
10.	.02634756	18.	5.73
11.	14136.226	19.	376940
12.	.0000004	22.	214441.6
13.	1043.6556	23.	2.24
14.	10603.92	24.	2056200
15.	564.23	25.	6809076

(Art. **157**, p. 105.)

8.	.0327	20.	186.57446+
9.	.176382	22.	.00054276
10.	.478	23.	.04608
12.	23600	24.	.0007842
14.	.5625	27.	.240575625
15.	1.370203125	28.	8.764
17.	1.25925+	29.	.01366703125
18.	19.55172+	30.	.0000029296375
19.	1.86574+		

(Art. **158**, p. 107.)

3.	.53125	6.	.5, .6, .078125, .12, .63398+, .48275+, .46200+
5.	.41176+		

(Art. **159**, p. 108.)

4. $\frac{4765}{10000}=\frac{953}{2000}$, $\frac{87698}{100000}=\frac{43849}{50000}$, $\frac{476}{1000000}=\frac{119}{250000}$, $\frac{75}{10000}=\frac{3}{400}$.

(ART. **160**, p. 108)

3.
$$\begin{array}{r|l} 24 & 15. \\ 20 & 18.625 \\ 12 & 6.93125 \\ \hline \end{array}$$
Ans. .57760+

4.
$$\begin{array}{r|l} 20 & 5. \\ 3 & 2.25 \\ 8 & 4.75 \\ 12 & 8.59375 \\ \hline \end{array}$$
Ans. .71614+

5.
$$\begin{array}{r|l} 16 & 4. \\ 16 & 8.25 \\ 25 & 21.515625 \\ 4 & 3.860625 \\ 20 & 12.96515625 \\ \hline \end{array}$$
Ans. .6482578125

7.
$$\begin{array}{r|l} 144 & 36. \\ 9 & 8.25 \\ 30\frac{1}{4} & 25.91666+ \\ 40 & 36\ 85674+ \\ 4 & 3.92141+ \\ 640 & 543.98035+ \\ \hline \end{array}$$
Ans. .84996+

8.
$$\begin{array}{r|l} 1728 & 1725. \\ 128 & 56.99826+ \\ \hline \end{array}$$
Ans. .44529+

9.
$$\begin{array}{r|l} 2\frac{1}{4} & 1. \\ 4 & 2.44444+ \\ 4 & 3.61111+ \\ \hline \end{array}$$
Ans. .90277+

10.
$$\begin{array}{r|l} 4 & 3. \\ 2 & 1.75 \\ 4 & 3.875 \\ \hline \end{array}$$
Ans. .96875

11.
$$\begin{array}{r|l} 2 & 1. \\ 8 & 7.5 \\ 4 & 3.9375 \\ \hline \end{array}$$
Ans. .984375

12.
$$\begin{array}{r|l} 60 & 30. \\ 60 & 40.5 \\ 24 & 0.675 \\ 7 & 6.028125 \\ 4 & 3.86116+ \\ \hline \end{array}$$
Ans. 96529+

13.
$$\begin{array}{r|l} 60 & 50.'' \\ 60 & 0.83333'+ \\ 30 & 24.01388^{\circ}+ \\ 12 & 8.80046+ \\ \hline \end{array}$$
Ans. .73337+

14.
$$\begin{array}{r|l} 16 & 8. \\ 25 & 0.5 \\ 4 & 0.02 \\ 20 & 19.005 \\ \hline \end{array}$$
Ans. .95025

15.
$$\begin{array}{r|l} 3 & 2. \\ 12 & 6.66666+ \\ 3 & 0.55555+ \\ 5\frac{1}{2} & 2.18518+ \\ 40 & 25.39730+ \\ 8 & 3.63493+ \\ \hline \end{array}$$
Ans. .45436+

(Art. **155**, p. 102.)

4. 832.670128
5. 869510.12305713
6. 53.854
7. 65999999.999934
8. .87218446
9. 874368.465731
10. 1.5200642

(Art. **156**, p. 103.)

9. 180.3774
10. .02634756
11. 14136.226
12. .0000004
13. 1043.6556
14. 10603.92
15. 564.23
16. 34672800000
18. 5.73
19. 376940
22. 214441.6
23. 2.24
24. 2056200
25. 6809076

(Art. **157**, p. 105.)

8. .0327
9. .176382
10. .478
12. 23600
14. .5625
15. 1.370203125
17. 1.25925+
18. 19.55172+
19. 1.86574+
20. 186.57446+
22. .00054276
23. .04608
24. .0007842
27. .240575625
28. 8.764
29. .01366703125
30. .0000029296375

(Art. **158**, p. 107.)

3. .53125
5. .41176+
6. .5, .6, .078125, .12, .63398+, .48275+, .46200+

(Art. **159**, p. 108.)

4. $\frac{4765}{10000}=\frac{953}{2000}$, $\frac{87698}{100000}=\frac{43849}{50000}$, $\frac{476}{1000000}=\frac{119}{250000}$, $\frac{75}{10000}=\frac{3}{400}$.

(Art. **160**, p. 108)

3.
$$\begin{array}{r|l} 24 & 15. \\ 20 & 18.625 \\ 12 & 6.93125 \\ \hline \end{array}$$
Ans. .57760+

4.
$$\begin{array}{r|l} 20 & 5. \\ 3 & 2.25 \\ 8 & 4.75 \\ 12 & 8.59375 \\ \hline \end{array}$$
Ans. .71614+

5.
$$\begin{array}{r|l} 16 & 4. \\ 16 & 8.25 \\ 25 & 21.515625 \\ 4 & 3.860625 \\ 20 & 12.96515625 \\ \hline \end{array}$$
Ans. .6482578125

7.
$$\begin{array}{r|l} 144 & 36. \\ 9 & 8.25 \\ 30\frac{1}{4} & 25.91666+ \\ 40 & 36.85674+ \\ 4 & 3.92141+ \\ 640 & 543.98035+ \\ \hline \end{array}$$
Ans. .84996+

8.
$$\begin{array}{r|l} 1728 & 1725. \\ 128 & 56.99826+ \\ \hline \end{array}$$
Ans. .44529+

9.
$$\begin{array}{r|l} 2\frac{1}{4} & 1. \\ 4 & 2.44444+ \\ 4 & 3.61111+ \\ \hline \end{array}$$
Ans. .90277+

10.
$$\begin{array}{r|l} 4 & 3. \\ 2 & 1.75 \\ 4 & 3.875 \\ \hline \end{array}$$
Ans. .96875

11.
$$\begin{array}{r|l} 2 & 1. \\ 8 & 7.5 \\ 4 & 3.9375 \\ \hline \end{array}$$
Ans. .984375

12.
$$\begin{array}{r|l} 60 & 30. \\ 60 & 40.5 \\ 24 & 0.675 \\ 7 & 6.028125 \\ 4 & 3.86116+ \\ \hline \end{array}$$
Ans. 96529+

13.
$$\begin{array}{r|l} 60 & 50.'' \\ 60 & 0.83333'+ \\ 30 & 24.01388^{\circ}+ \\ 12 & 8.80046+ \\ \hline \end{array}$$
Ans. .73337+

14.
$$\begin{array}{r|l} 16 & 8. \\ 25 & 0.5 \\ 4 & 0.02 \\ 20 & 19.005 \\ \hline \end{array}$$
Ans. .95025

15.
$$\begin{array}{r|l} 3 & 2. \\ 12 & 6.66666+ \\ 3 & 0.55555+ \\ 5\frac{1}{2} & 2.18518+ \\ 40 & 25.39730+ \\ 8 & 3.63493+ \\ \hline \end{array}$$
Ans. .45436+

(ART. **161**, p. 110.)

2. .9375gal.
4
3.7500qt.
2
1.50pt.
4
2.0gi.

Ans. 3qt. 1pt. 2gi.

4. .7694a.
4
3.0776r.
40
3.1040rd.
30¼
26
3120
3.146yd.
9
1.314ft.
144
1256
1256
314
45.216in.

Ans. 3r. 3rd. 3yd. 1ft. 45.216in.

3. .84 l.m.
4
3.36wk.
7
2.52d.
24
208
104
12.48h.
60
28.80m.
60
48.0sec.

Ans. 3wk. 2d. 12h. 28m. 48sec.

5. .6543m.
8
5.2344fur.
40
9.3760rd.
5½
188
1880
2.068yd.
3
.204ft.
12
2.448in.
3
1.344b.c.

Ans. 5fur. 9rd. 2yd. 2in. 1.344b.c.

6.
.54324lb.
12
6.51888oz.
20
10.37760dwt.
24
15104
7552
9.0624gr.

Ans. 6oz. 10dwt. 9.0624gr.

7.
.5769lb.
12
6.9228oz.
8
7.3824dr.
3
1.1472sc.
20
2.9440gr.

Ans. 6oz. 7dr. 1sc. 2.944gr.

9.
.9876m.
8
7.9008fur.
10
9.0080ch.
100
.8 li.
7.92
6.336in.

Ans. 7fur. 9ch. 6.336in.

8.
.0876t.
20
1.7520cwt.
4
3.008qr.
25
.200lb.
16
3.2oz.
16
3.2dr.

Ans. 1cwt. 3qr. 3oz. 3.2dr.

10.
.4698c.
8
3.7584c.ft.
16
45504
7584
12.1344cu.ft.
1728
10752
2688
9408
1344
232.2432c.in.

Ans. 3c.ft. 12cu.ft. 232.2432c.in.

11. .8694yd.
4
3.4776qr.
4
1.9104na.
$2\frac{1}{4}$
2276
18208
2.0484in.

Ans. 3qr. 1na. 2.0484in.

12. .7564bush.
4
3.0256pk.
8
.2048qt.
2
.4096pt.

Ans. 3pk. .4096pt.

MISCELLANEOUS EXAMPLES IN DECIMAL FRACTIONS.

(Art. 162, p. 110.)

1. 14c. × 14.75 = \$2.065, Ans.
2. \$12.375 × 20.5 = \$253.6875 = \$253.687½, Ans.
3. \$62.50 × 67.75 = \$4234.375, Ans.
4. \$4234.375 ÷ 67.75 = \$62.50, Ans.
5. \$4234.375 ÷ \$62.50 = 67.75, Ans.
6. \$75 — \$62.50 = \$12.50; \$846.875 ÷ \$12.50 = 67.75, Ans.
7. \$5081.25 ÷ 67.75 = \$75; \$75 — \$62.50 = \$12.50, gain per acre; \$12.50 × 67.75 = \$846.875, total gain.

8. \$12.50
87½c. × 3 = 2.62½
8½c. × 24.5 = 2.08¼
37½c. × 3 = 1.12½
62½c. × 2 = 1.25
11c. × 6 = .66
4¼c. × 15 = .63¾
22c. × 4 = .88
Ans. \$21.76

9.

t.	cwt.	qr.	lb.		t.
2	15	0	12½	=	2.75625
1	19	0	6¼	=	1.953125
2	4	0	18¾	=	2.209375
3	0	0	0	=	3.
Total weight,					9.918750

\$15.875 × 9.91875 = \$157.46$\frac{1}{64}$, Ans.

10. \$1.875 — \$1.375 = 50c., gain per head, 1st Ans.
50c. × 100 = \$50, total gain, 2d Ans.

11. 33yd. + 50yd. = 83yd.; 133.5yd. — 83yd. = 50.5yd.;
3.33\frac{1}{3}$ × 33 = $110.
$3.875 × 50 = $193.75
$3.60 × 50.5 = $181.80
$485.55, Total receipts.
$3.25 × 133.5 = $433.875, Total cost.
$51.675, Net gain, Ans.

12. 13yd. 2qr. 3na. = 54.75qr.; 54.75 ÷ 6 = 9.125, No. ells. $4.67 × 9.125 = $42.61375 = 42.61\frac{3}{8}$, Ans.

13. $11.75 × 6.375 × 7.8 = $584.26875 = 584.26\frac{7}{8}$, Ans.

14. 37$\frac{1}{2}$c. × 356.25 = $133.59375, cost of wool. $133.59375 + $62.50 + $37.50 = $233.59375 = 233.59\frac{3}{8}$, Ans.

15. $15.625 × 5.625 = $87.890625 = 87.89\frac{1}{16}$, Ans.

Explanation. — Had the price been $15.625 per *foot*, the cost would have been $15.625 × 5625; but since the price is $15.625 per *thousand* feet, the true multiplier is *one thousandth part* of 5625, viz. 5.625.

16. 43a. 3r. 25rd. = 43.90625a.; $62.875 × 43.90625 = $2760.60546875 = 2760.60\frac{35}{64}$, Ans.

17. 3t. 15cwt. 2qr. 12$\frac{1}{2}$lb. = 3.78125t.; $9.75 × 3.78125 = $36.8671875 = 36.86\frac{23}{32}$, Ans.

18. 43m. 7fur. 25rd. = 43.953125m.; $9562.875 × 43.953125 = $420318.240234375, Ans.

19. (20 + 15) × 2 × 8.5 = 595, No. sq. ft. in 4 walls; 8 × $\frac{5}{8}$ × 9 = 45, No. sq. ft. in 1 roll; 595 ÷ 45 = 13$\frac{2}{9}$, No. rolls; 62$\frac{1}{2}$c. × 13$\frac{2}{9}$ = 8.263\frac{8}{9}$, Ans.

20. .875cwt. = .04375t.; 2£ 3s. 6d. 1qr. = 2$\frac{169}{960}$£; 2$\frac{169}{960}$£ × .04375 = .09520$\frac{35}{192}$£ = 1s. 10d. 3.39375qr., Ans.

21. (63.5rd. + 27.75rd.) × 2 = 182.5rd., distance round; 87$\frac{1}{2}$c. × 182.5 = $159.6875 = 159.68\frac{3}{4}$, Ans.

COMPOUND ADDITION.

(ART. **165**, p. 113.)

3. 49£ 9s. 7d. 2qr.
5. 46 lb, 1oz. 15dwt. 12gr.
7. 118yd. 2qr. 1½in.
8. 93gal. 3qt. 2gi.
9. 189a. 3r. 36rd.
10. 213a. 3r. 32rd. 4yd. 2ft. 80in.
11. 333m. 2fur. 17 li. 6$\frac{2}{25}$in.
12. 58yd. 3na. 1¾in.
13. 16circ. 10s. 24° 37′ 30″.
14. 234bush. 2pk. 5qt. 1pt.
15. 32t. 6cwt. 2qr. 5lb.

COMPOUND SUBTRACTION.

5. 2t. 15cwt. 3qr. 4lb. 1oz. 14dr.
6. 12lb. 7oz. 1dr. 15gr.
9. 7m. 1fur. 4rd. 5yd. 2ft.
10. 2circ. 331° 69m. 4fur. 39rd. 5yd. 2ft.

13.
$$\left.\begin{array}{cccccc} \text{a.} & \text{r.} & \text{rd.} & \text{yd.} & \text{ft.} & \text{in.} \\ 12 & 3 & 0 & 0 & 5 & 0 \\ 12 & 2 & 39 & 30 & 6 & 36 \end{array}\right\} = \left\{\begin{array}{cccccc} \text{a.} & \text{r.} & \text{rd.} & \text{yd.} & \text{ft.} & \text{in.} \\ 12 & 2 & 39 & 30 & 7 & 36 \\ 12 & 2 & 39 & 30 & 6 & 36 \\ \hline & & \text{Ans.} & & 1 & \end{array}\right.$$

14.
$$\left.\begin{array}{cccccccc} \text{circ.} & \text{deg.} & \text{m.} & \text{fur.} & \text{rd.} & \text{yd.} & \text{ft.} & \text{in.} \\ 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 359 & 69 & 3 & 39 & 5 & 1 & 5 \end{array}\right\} = \left\{\begin{array}{cccccccc} \text{circ.} & \text{deg.} & \text{m.} & \text{fur.} & \text{rd.} & \text{yd.} & \text{ft.} & \text{in.} \\ 9 & 359 & 69 & 3 & 39 & 5 & 1 & 6 \\ 9 & 359 & 69 & 3 & 39 & 5 & 1 & 5 \\ \hline & & & & & \text{Ans.} & & 1 \end{array}\right.$$

16. 8yr. 3m. 27d.
17. 1yr. 1m. 9d.
18. 4yr. 2m. 3d.
19. 1yr. 0m. 7d.
20. 11yr. 1m. 10d.
21. 39yr. 5m. 23d.
22. 111yr. 11m. 8d.
23. 39yr. 2m. 7d.

(ART. **171**, p. 118.)

		a.	r.	rd.			m.	fur.	rd.
1.	Minuend,	108	3	17	2.		187	7	19
		15	2	29			53	3	14
		11	0	33			71	0	18
	Subtrahend,	26	3	22			124	3	32
	Ans.	81	3	35		Ans.	63	3	27

		a.	r.	rd.		£	s.	d.	
3.	Bought	723	0	0	for	963	14	9	
		253	3	17		319	12	0	
		176	0	14		237	0	11	
	Sold	429	3	31	for	556	12	11	
	1st Ans.	293	0	9		407	1	10,	2d Ans.

4. Variable.

	t.	cwt.	qr.	lb.
5.	11	7	3	18
	6	19	0	7
		18	3	0
	17	0	3	0
Min.	36	6	2	0
	12	7	0	12
	8	15	3	0
Sub.	21	2	3	12
Ans.	15	3	2	13

	cwt.	qr.	lb.
6.	17	3	8
	5	3	18
	5	3	18
Beef,	29	2	19
	3	2	12
	4	1	18
	5	3	6
Pork,	13	3	11
Ans.	15	3	8

	rd.	yd.	ft.	in.	b.o.		rd.	yd.	ft.	in.	b.o.
7.	7	2	0	2	1						
	2	3	1	3	2						
Sum,	9	5	1	6	0	=	9	5	1	6	0
	14	0	2	7	1						
	4	0	2	7	2						
Difference,	9	4½	2	11	2	=	9	5	1	5	2
									Ans.		1

	yd.	qr.	na.	in.
8. Min.	18	3	3	2
	4	0	3	0
	3	2	0	0
	5	1	2	1
Sub.	13	0	1	1
Ans.	5	3	2	1

	gal.	qt.	pt.	gi.
9.	65	3	1	0
	74	0	1	3
Min.	140	0	0	3
Sub.	83	1	0	0
Ans.	56	3	0	3

10. 1yr. 2m. 15d.

11. 82yr.

COMPOUND MULTIPLICATION.

7. 530 lb. 2oz. 3dr. 2sc. 8gr.
8. 958m. 5fur. 5ch. 12 li. $5\frac{12}{25}$in.
10. 1644yd. 2qr. 2na.
11. 1742gal. 3qt. 1pt. 3gi.
13. 187c. 2c. ft. 5cu. ft.; \$1061.92$\frac{15}{128}$.
14. 2s. 4° 59′ 3″.
15. 124rd. 2ft. 6in.

COMPOUND DIVISION.

2. 9rd. 4yd. 2ft. 6in. 2b.c.
3. 2gal. 3qt. 1pt. 2gi.
4. 4t. 3cwt. 2qr. 14 lb. 8oz. 4dr.
5. 5a. 3r. 37rd. 25yd. 8ft.
7. 8 lb. 4oz. 7dr. 2sc. 16gr.
8. 9m. 7fur. 8ch. 3rd. 15 li. 6in.
10. 27yd. 3qr. 2na.
11. 17gal. 3qt. 1pt. 3gi.
12. 3£ 15s. 6d. 1qr.
13. 1c. 3c. ft. 7cu. ft.; \$5.67.
14. 2° 14′ 27″.
15. 7rd. 5ft.

DUODECIMALS.

2.	14	8′	3″				14	8′	3″			
	13	6′	9″				13	6′	9″			
	190	11′	3″				182	104′	39″			
	7	4′	1″	6‴				84′	48″	18‴		
		11′	0″	2‴	3⁗				126″	72‴	27⁗	
	199	2′	4″	8‴	3⁗	=	199	2′	4″	8‴	3⁗	

Note.—Each partial product may be reduced, as in the first operation above, or, if preferred, the plan of the second operation may be adopted, in which only the several *sums* are reduced.

3. 8ft. 9′ 3″ × 3ft. 2′ 4″ × 2ft. 5′ 7″ = 69ft. 0′ 10″ 4‴ 5⁗ 1′′′′′, Ans.

4. 40ft. 8′ × 15ft. 6′ × 1ft. 4′ = 840ft. 5′ 4″; 840ft. 5′ 4″ × 27 = 22692, Ans.

NOTE.—The number of bricks in a cubic foot is 27. This may be found by dividing the number of inches in a foot, 1728, by the number of inches in a brick, 8 × 4 × 2 = 64; thus, 1728 ÷ 64 = 27; but the intelligent pupil will be more interested in, and will receive important ideas from, some plan similar to that given in the following dialogue, which the teacher will readily illustrate by means of two books. *Teacher.* A brick is 8 inches long, then how many bricks, lying end to end, will extend a foot? *Pupil.* 1½. *T.* A brick is 4 inches wide, how many bricks, lying side by side, will extend a foot? *P.* 3. *T.* Then how many bricks, lying flatwise, will cover over a square foot? *P.* 3 times 1½ = 4½. *T.* A brick is 2 inches thick, how many bricks, lying one upon another, will extend up a foot? *P.* 6. *T.* Then how many bricks in a cubic foot? *P.* 6 times 4½ = 27.

5. (38 + 24) × 2 = 124ft., length of the 4 walls. 124ft. × 18ft. 6′ × 1ft. 4′ = 3058ft. 8′; 3058ft. 8′ × 27 = 82584, Ans.

NOTE.—If the side walls are considered to be 38ft. long, then each end wall must be considered twice 1ft. 4′ = 2ft. 8′ *shorter* than the width of the house; ∴ 26ft. 8′ — 2ft. 8′ = 24ft., is the length of each end wall.

6.

$$\begin{array}{rl} 5 & 8' \\ 5 & 4' \\ 4 & 9' \\ 4 & 3' \\ \hline 20 & 0', \text{ height of 4 windows.} \end{array}$$

3ft. 9′ × 20 = 75ft., area of 4 windows.
75ft. × 11 = 825ft., Ans.

7. 12ft. 4′ × 2ft. 3′ × 4 = 111ft., Ans.

8. 278ft. 5′ 3″. 9. 179ft. 1′ 10″ 6‴.

$$\begin{array}{lrrrcrrr} & \multicolumn{3}{c}{(10.)} & & \multicolumn{3}{c}{(11.)} \\ & 5 & 8' & & & 5 & 6' & \\ & 4 & 11' & & & 4 & 9' & \\ \cline{2-3}\cline{6-7} & 22 & 8' & & & 22 & 0' & \\ & 5 & 2' & 4'' & & 4 & 1' & 6'' \\ \cline{2-4}\cline{6-8} \text{Top,} & 27 & 10' & 4'' & & 26 & 1' & 6'' \\ & & & 2 & & & & 2 \\ \cline{2-4}\cline{6-8} \text{Top and bottom,} & 55 & 8' & 8'' & & 52 & 3' & 0'' \\ & \multicolumn{3}{c}{\text{(continued.)}} & & \multicolumn{3}{c}{\text{(continued.)}} \end{array}$$

	(10 continued.)			(11 continued.)		
	5	8′		5	6′	
	3	8′		3	6′	
	17	0′		16	6′	
	3	9′	4″	2	9′	0″
One side,	20	9′	4″	19	3′	
			2		2	
Two sides,	41	6′	8″	38	6′	
	4	9′		4	7′	
	3	8′		3	6′	
	14	3′		13	9′	
	3	2′	0″	2	3′	6″
One end,	17	5′		16	0′	6″
		2				2
Two ends,	34	10′		32	1′	0″
Top and bottom,	55	8′	8″	52	3′	
Two sides,	41	6′	8″	38	6′	
Two ends,	34	10′	0″	32	1′	
One box,	132	1′	4″	122	10′	
			12		12	
Ans.	1585	4′	0″	1474	0′	
	1474	0′				
	111	4′, Ans. to Ex. 11.				

12. 1474ft. ÷ 12 = 122ft. 10′, Ans.

Explanation.—Divide the number of feet of boards required to make the smaller set of boxes by 12, because 12 feet of boards make a cubic foot. See explanation of Ex. 32, p. 30 of the Key.

13. (18ft. 6′ + 15ft. 9′) × 2 = 68ft. 6′, length of 4 walls.
68ft. 6′ × 8ft. 4′ = 570ft. 10′, area of 4 walls.
18ft. 6′ × 15ft. 9′ × 2 = 582ft. 9′, area of floor and ceiling.
1153ft. 7′ Ans.

14. 16ft. 6′ × 15 ft. 8′ ÷ 9 = 28yd. 6ft. 6′, Ans.
15. 38ft. 6′ × 24ft. 8′ × 7ft. 9′ = 7359ft. 11′, Ans.
16. 45ft. 6 × 29ft. × 6ft. 6′ ÷ 27 = 317yd. 17ft. 9′, Ans.
18. 20ft. 6′ × 8ft. ÷ 32 = 5c. 1c.ft., Ans.
19. 67ft. 9′ × 17ft. 3′ ÷ 32 = 36c. 4c.ft. 2cu.ft. 9′, Ans.
20. 25ft. × 7ft. ÷ 32 = 5c. 3c.ft. 12cu.ft., Ans.
21. 33ft. 8′ × 6ft. 6′ × 3ft. 10′ ÷ 128 = 6c. 4c.ft. 6cu.ft. 10′ 4″, Ans.

INTEREST.

6.	$.497⅚	19.	$1.20848
7.	$.297	20.	$.4595
8.	$.871⅙	21.	$.195676
9.	$.125⅚	22.	$4.175860⅛
10.	$.220½	30.	$217.106
15.	$8.0788	31.	$10.554
16.	$12.36172⅔	32.	$.228
17.	$3.06175	33.	$388.996
18.	$3.16309	34.	$19.246

	yr.	m.	d.	
35.	1854	2	11	
	1851	6	7	$125 × .160⅔ = $20.083, Ans.
	2	8	4	
36.	1855	1	26	
	1852	4	18	$76.72 × .166⅓ = $12.761, Ans.
	2	9	8	
37.	1858	9	19	
	1854	8	17	$1728 × .245⅓ = $423.936, Ans.
	4	1	2	
38.	1857	12	30	
	1853	10	12	$111.111 × .253 = $28.111, Ans.
	4	2	18	

	yr.	m.	d.	
39.	1841	5	12	
	1816	11	13	$37.75 × 1.469⅚ = $55.486, Ans.
	24	5	29	
40.	1842	6	21	
	1818	3	26	$100 × 1.454⅙ = $145.417, Ans.
	24	2	25	

41. 25£ 15s. 8d. 3qr. = 25.786£;
25.786£ × .167½ = 4.319155£ = 4£ 6s. 4d. 2qr., Ans.

42. 144£ 15s. 8d. 2qr. = 144.785£;
144.785£ × .093 = 13.465005£ = 13£ 9s. 3d. 2qr., Ans.

43. 75£ 6s. 4d. 1qr. = 75.318£;
75.318£ × .236 = 17.775048£ = 17£ 15s. 6d., Ans.

44. 87£ 10s. 9d. = 87.538£;
87.538£ × .238⅙ = 20.848633£ = 20£ 16s. 11d. 3qr., Ans.

45. 100£ 18s. 3qr. = 100.903£;
100.903£ × .24 = 24.21672£ = 24£ 4s. 4d., Ans.

49. $4698.72 = interest for 60 days.
9397.44 = " 120 days; 120 = 60 × 2.
234.936 = " 3 days; 3 = $\frac{1}{20}$ of 60.
$9632.376 = " 123 days, Ans.

50. $7.6487 = interest for 60 days.
.3824 = " 3 days; 3 = $\frac{1}{20}$ of 60.
$8.0311 = " 63 days, Ans.

51. $4.68 = interest for 60 days.
2.34 = " 30 days; 30 = ½ of 60.
1.17 = " 15 days; 15 = ½ of 30.
.234 = " 3 days; 3 = $\frac{1}{10}$ of 30.
$3.744 = " 48 days;
$468 — $3.744 = $464.256, Ans.

52. \$8.4428 = interest for 60 days.
4.2214 = " 30 days; 30 = $\frac{1}{2}$ of 60.
.4221 = " 3 days; 3 = $\frac{1}{10}$ of 30.
\$13.0863 = " 93 days;
\$844.28 — \$13.086 = \$831.194, Ans.

53. \$23.48 = interest for 60 days.
11.74 = " 30 days; 30 = $\frac{1}{2}$ of 60.
1.174 = " 3 days; 3 = $\frac{1}{10}$ of 30.
\$12.914 = " 33 days;
\$2348 — \$12.914 = \$2335.086, Ans.

55. \$856.47 × .106$\frac{1}{3}$ = \$91.07131, interest at 6 per cent.; \$91.07131 ÷ 6 = \$15.17855; \$15.17855 × 7 = \$106.25, Ans.

56. \$666.666 × .391 = \$260.6664, interest at 6 per cent.; \$260.6664 ÷ 3 = \$86.8888; \$86.8888 × 4 = \$347.555, Ans.

NOTE.—The pupil will readily see that to divide by 3 and multiply by 4 is, in effect, the same as to divide by 6 and multiply by 8. It is desirable to use the smallest numbers that will represent the proper relations.

57. \$358.14 × .295$\frac{1}{3}$ = \$105.77068, interest at 6 per cent.; \$105.77068 ÷ 6 = \$17.62844; \$17.62844 × 9$\frac{1}{2}$ = \$167.47, Ans.

58. \$33.45 × .108 = \$3.6126, interest at 6 per cent.; \$3.6126 ÷ 3 = \$1.2042; \$1.204 × 2 = \$2.408, Ans.

59. \$500 × .109 = \$54.50, interest at 3 per cent.; \$500 + \$54.50 = \$554.50, Ans.

60.	\$52.241	64.	\$2.25
61.	\$252.135	65.	\$1033.346
62.	\$148.139	66.	\$2714.806
63.	\$2.25	67.	\$825.139

(ART. **203**, p. 138.)

3.

yr.	m.	d.
1855	3	26
1854	11	13
1853	11	11
1853	10	15
1851	7	14
1850	9	6
1850	6	4
	3	2
	10	8
2	3	1
		26
1	0	2
	4	13

NOTE. — This is a convenient manner of solving examples in partial payments. *Every figure made in solving the example is preserved, and the work may be examined at any stage of the operation.* In finding the several *times*, the *minuend* for the *first* subtraction is the *subtrahend* for the *second*, the minuend for the *second* is the subtrahend for the *third*, etc., and the several remainders are written *under* each other, *in the order in which they were found.* The first principal is added to the interest by merely *prefixing* 10 to the interest. This mode will often shorten the work. A mere *inspection* shows that the third and fourth payments are not so large as the interests accrued at the times of their payments, severally.

The sum of the 3d, 4th, and 5th payments is found below.

3d Payment,	$25
4th Payment,	28
5th Payment,	125
	$178

$1000.
.015$\frac{1}{3}$
333
15000
1015.333
50.
965.333
.051$\frac{1}{3}$
321777
965333
4826665
49.553760
965.333
1014.887
150.
864.887
.135$\frac{1}{6}$
144147
4324435
2594661
864887
116.903892
864.887
981.791
178
803.791
.004$\frac{1}{3}$
267930
3215164
3.483094
803.791
807.274
75.
732.274
.060$\frac{1}{3}$
244091
43936440
44.180531
732.274
776.455
500.
276.455
.022$\frac{1}{6}$
46075
552910
552910
6.128085
276.455
$282.583, Ans.

(4.)

$756.75	Principal.
31.027	Int. from Jan. 12, '52, to Sept. 18, '52; 8m. 6d.
787.777	Amount of Principal to Sept. 18, 1852.
300.777	1st Payment.
487.	1st Remainder, forming the 2d Principal.
37.499	Int. from Sept. 18, '52, to Dec. 30, '53; 1yr. 3m. 12d.
524.499	Amount of 2d Principal to Dec. 30, 1853.
300.499	2d Payment.
224.	2d Remainder, forming the 3d Principal.
17.696	Int. from Dec. 30, '53, to Apr. 24, '55; 1yr. 3m. 24d.
241.696	Amount of 3d Principal to Apr. 24, 1855.
61.696	Sum of 3d and 4th Payments.
180.	3d Remainder, forming the 4th Principal.
7.38	Int. from Apr. 24, '55, to Dec. 30, '55; 8m. 6d.
$187.38	Amount due Dec. 30, 1855, Ans.

(5.)

$500.	Principal.
30.	Int. from July 1, '51, to July 1, '52; 1yr.
530.	Amount of Principal to July 1, 1852.
350.	1st Payment.
180.	1st Remainder, forming 2d Principal.
5.40	Int. from July 1, '52, to Jan. 1, '53; 6m.
$185.40	Amount due Jan. 1, 1853, Ans.

NOTE.—From the nature of the example, interest commences July 1, 1851, six months after the date of the Note.

(6.)

$3476.875	Principal.
45.199	Int. from July 18, '49, to Oct. 6, '49; 2m. 18d.
3522.074	Amount of Principal to Oct. 6, 1849.
747.56	1st Payment.
2774.514	1st Remainder, forming the 2d Principal.
45.779	Int. from Oct. 6, '49, to Jan. 15, '50; 3m. 9d.
2820.293	Amount of 2d Principal to Jan. 15, 1850.
54.75	2d Payment.
2765.543	2d Remainder, forming the 3d Principal.
485.353	Int. from Jan. 15, '50, to Dec. 18, '52; 2yr. 11m. 3d.
3250.896	Amount of 3d Principal to Dec. 18, 1852.
1043.32	Sum of 3d and 4th Payments.
2207.576	3d Remainder, forming the 4th Principal.
43.416	Int. from Dec. 18, '52, to April 16, '53; 3m. 28d.
2250.992	Amount of 4th Principal to Apr. 16, 1853.
1000.	5th Payment.
1250.992	4th Remainder, forming the 5th Principal.
97.577	Int. from Apr. 16, '53, to Aug. 4, '54; 1yr. 3m. 18d.
$1348.569	Amount due Aug. 4, 1854, Ans.

(7.)

$448.50	Principal.
14.913	Int. from June 15, '54 to Dec. 6, '54; 5m. 21d.
463.413	Amount of Principal to Dec. 6, 1854.
75.	1st Payment.
388.413	1st Remainder, forming the 2d Principal.
10.045	Int. from Dec. 6, '54, to Apr. 19, '55; 4m. 13d.
398.458	Amount of 2d Principal to Apr. 19, 1855.
125.	2d Payment.
273.458	2d Remainder, forming the 3d Principal.
14.676	Int. from Apr. 19, '55, to Jan. 25, '56; 9m. 6d.
288.134	Amount of 3d Principal to Jan. 25, 1856.
110.	Sum of 3d and 4th Payments.
178.134	3d Remainder, forming the 4th Principal.
5.473	Int. from Jan. 25, '56 to July 3, '56; 5m. 8d.
$183.607	Amount due July 3, 1856, Ans.

(Art. **204**, p. 140.)

(2.)

Principal,		$1250.75
Int. of Principal to May 15, '55; 11m. 11d.,		71.084
Amount of Principal to May 15, 1855,		$1321.834
1st Payment,	$300.	
Int. of 1st Payment from Dec. 16, 4m. 29d.,	7.45	
2d Payment,	250.	
Int. of 2d Payment from Jan. 1, 4m. 14d.,	5.583	
3d Payment,	400.	
Int. of 3d Payment from Feb. 18, 2m. 27d.,	5.80	
4th Payment,	300.	
Int. of 4th Payment from March 15, 2m.,	3.	
Sum of Payments with their Interest,		$1271.833
Sum due May 15, 1855, Ans.		$50.001

(Art. **207—210**, p. 141.)

2. $370 × .03 = $11.10; $55.50 ÷ $11.10 = 5, Ans.
3. $97 × .05 = $4.85; $33.95 ÷ $4.85 = 7, Ans.
4. $50 × .03½ = $1.75; $5.60 ÷ $1.75 = 3⅕, Ans.
5. $75 × .04 = $3.00; $27 ÷ $3 = 9, Ans.
7. $133 × .07 = $9.31; $32.585 ÷ $9.31 = 3.5, i.e. 3yr. 6m., Ans.
8. $50 × .09 = $4.50; $11.85 ÷ $4.50 = 2.63⅓, i.e. 2yr. 7m. 18d., Ans.
9. $150 × .06 = $9; $195 — $150 = $45; $45 ÷ $9 = 5, i.e. 5yr., Ans.
10. $56 × .08½ = $4.76; $69.09 — $56 = $13.09; $13.09 ÷ $4.76 = 2.75, i.e. 2yr. 9m., Ans.
11. $1000 × .09 = $90; $495 ÷ $90 = 5.5, i.e. 5yr. 6m., Ans.
12. Assume any sum, say $100; then the question is, in what time will $100 *gain* $100? Hence, $100 × .06 = $6; $100 ÷ $6 = 16⅔, i.e. 16yr. 8m., Ans. Hence, To find the time in which any sum will double itself, at any rate per cent, *divide* 100 *by the rate, and the quotient will be the time in years.*

13. Assume \$100; in what time will \$100 *gain* \$300?
\$100 × .05 = \$5; \$300 ÷ \$5 = 60; i.e. 60yr., Ans.
15. \$15 ÷ .025 = \$600, Ans.
16. \$100 ÷ .04 = \$2500, Ans.
17. \$1500 ÷ .06 = \$25000, Ans.
18. \$1600 ÷ .08 = \$20000, Ans.
19. \$74.32 ÷ .1065 = \$697.84, Ans.
20. \$59.76 ÷ .1045 = \$571.866, Ans.
22. \$360.585 ÷ 1.08 = \$333.875, Ans.
23. \$472 ÷ 1.18 = \$400; \$400 × .18 = \$72, Ans.
24. \$3569.47 ÷ 1.153 = \$3095.811, Ans.
25. \$480 ÷ 1.2 = \$400; \$400 × .20 = \$80, Ans.

COMPOUND INTEREST.

3. \$1000
1.07
7000
1000
1070.00
1.07
7490
1070
1144.90
1.07
801430
114490
1225.0430
1000. , 1st Principal.
\$225.043, Ans.

NOTE. — Multiplying by 1.07 is, in effect, the same as multiplying by .07 and *adding* the principal.

4. \$5000
1.06
30000
5000
5300.00
1.06
31800
5300
5618.00
1.06
33708
5618
5955.08
1.06
3573048
595508
6312.3848 [for 10m.
1.05, Amount of \$1
315619240
63123848
\$6628.004, Ans.

5.

$$\begin{array}{lr}
 & \$900 \\
\text{Amount of \$1 for 3m.} = & 1.015 \\ \hline
 & 4500 \\
 & 900 \\
 & 900 \\ \hline
 & 913.500 \\
 & 1.015 \\ \hline
 & 45675 \\
 & 9135 \\
 & 9135 \\ \hline
 & 927.203 \\
 & 1.015 \\ \hline
 & 4636015 \\
 & 927203 \\
 & 927203 \\ \hline
 & 941.111 \\
 & 1.015 \\ \hline
 & 4705555 \\
 & 941111 \\
 & 941111 \\ \hline
 & 955.228 \\
 & 1.015 \\ \hline
 & 4776140 \\
 & 955228 \\
 & 955228 \\ \hline
 & 969.556 \\
 & 1.015 \\ \hline
 & 4847780 \\
 & 969556 \\
 & 969556 \\ \hline
 & 984.099 \\
 & 1.015 \\ \hline
 & 4920495 \\
 & 984099 \\
 & 984099 \\ \hline
 & 998.860 \quad \text{Carried up.}
\end{array}$$

$$\begin{array}{lrl}
\text{Brought up,} & 998.860 & \\
 & 1.015 & \\ \hline
 & 4994300 & \\
 & 998860 & \\
 & 998860 & \\ \hline
 & 1013.843 & \\
 & 1.015 & \\ \hline
 & 5069215 & \\
 & 1013843 & \\
 & 1013843 & \\ \hline
 & 1029.051 & \\
 & 1.015 & \\ \hline
 & 5145255 & \\
 & 1029051 & \\
 & 1029051 & \\ \hline
 & 1044.487 & [\$1 \text{ for 1m.} \\
 & 1.005 & = \text{Amount of} \\ \hline
 & 5222435 & \\
 & 1044487 & \\ \hline
 & \$1049.709, \text{ Ans.} &
\end{array}$$

6. 48£ 10s. = 48.5£
1.06
2910
485
51.410
1.06
30846
5141
54.495
1.06
326970
54495
57.765
1.048
462120
231060
57765
60.538
48.5
12.038£
20
.760s.
12
9.12d.
4
.48qr.

Ans. 12£ 9d.

7. $3476.95
1.07
2433865
347695
3720.337
1.07
26042359
3720337
3980.761
.0395 Int. at 6 per [cent.
19903805
35826849
11942283
6)157.2400595
26.20667
7
183.4466
3980.761
4164.208
3476.95
$687.258, Ans.

8. $.278
9. $27.697
10. $30.599
11. $1462493.206

NOTE. — In the following examples in Compound Interest, it will be observed that the *interest* or the *amount* of $1 for a given time is taken from the Table, as seems most convenient; thus, in Ex. 12, solved in the Arithmetic, the *interest* is taken; also in Ex. 16, solved in the Key. But in the other examples, where the time is not for entire years or periods, it is necessary to use the amounts. Also where the amount is called for, as in Ex. 14.

15. $3.996019 = Amount of $1 at 8 per cent. for 18yr.
$1.049\frac{1}{3}$ = Amount of $1 at 8 per cent. for 7m. 12d.
1332006
35964171
15984076
3996019
4.193155937 = Amount of $1 at 8 per cent. for 18yr. [7m. 12d.
350.50 Principal.
209657796850
20965779685
12579467811
$1469.701, Ans.

16. $2.106849 { Amount of $1 at 4 per cent. for 19yr., or at 8 per cent. for 9yr. 6m., compounding semi-annually.
500
1053.424500 Amount of $500.
500. Principal.
$553.425, Ans.

17. $1731.676 Amount of $1000 at 4 per cent. for 14yr.
$1.005\frac{1}{3}$ Amount of $1 at 16 per cent. for 12d.
577225
8658380
1731676
$1740.911605, Ans.

COMMISSION.

1. $1350 × .02 = $27.00, Ans.
2. $5725 × .03 = $171.75, Ans.
3. $18000 × .002 = $36, Ans.
4. $6350 × $.00\frac{1}{4}$ = $$15.87\frac{1}{2}$, Ans.
5. $3484.50 × .01 = $34.845, Ans.

6. 75c. × 100 × 34 = $2550; $2550 × .01½ = $38.25, 1st Ans.
$2550 — $38.25 = $2511.75, 2d Ans.
7. 500cwt. 3qr. 12½ lb. = 500.875cwt.
$9 × 500.875 = $4507.875, cost of sugar.
$4507.875 × .03½ = $157.776, Ans.
8. 37½c. × 3560 = $1335; $1335 × .02 = $26.70, 1st Ans.
$1335 — $26.70 = $1308.30, 2d Ans.

INSURANCE.

1. $5000 × .002 = $10; $10 + $1.25 = $11.25, Ans.
2. $500000 × .02 = $10000, Ans.
3. $21500 × .03¼ = $698.75, Ans.
4. 534£ 10s. 6d. = 534.525£; $4.87 × 534.525 = $2603.137; $2603.137 × .01¾ = $45.555; $45.555 + $1.25 = $46.805, Ans.
5. $12000 × .0097 = $116.40, Ans.
6. $5000 × .0109 = $54.50, Ans.
7. $3000 × .005 = $15, Ans.
8. 437£ 16s. 3d. = 437.8125£; $4.87 × 437.8125 = $2132.147; $2132.147 × .02½ = $53.304, Ans.

DISCOUNT.

2. $756 ÷ 1.08 = $700, Ans.
3. $475 ÷ 1.033 = $459.826; $475 — $459.826 = $15.174, Ans.
4. $1800 ÷ 1.095 = $1643.836, 1st Ans.
$1800 — $1643.836 = $156.164, 2d Ans.
5. 1856 2 20 $850 ÷ 1.042 = $815.739, Ans.
1855 6 8
8 12 = Term of credit.
6. $800 ÷ 1.05 = $761.905, Ans.

7. $\$256 \div 1.18 = \$216.949$; $\$256 - \$216.949 = \$39.051$, Ans.

8. $\$600 \div 1.04 = \$576.923$, Ans.

9. $$\begin{array}{rrr} 1858 & 5 & 1 \\ 1857 & 6 & 20 \\ \hline & 10 & 11 \end{array} = \text{Term of credit.}$$ $\$1000 \div 1.051\frac{5}{6} = \$950.721$, Ans.

10. $\$325.75 \div 1.032 = \$315.649$; $\$325.75 - \$315.649 = \$10.101$, Ans.

11. $$\begin{array}{rrr} 1857 & 6 & 24 \\ 1857 & 5 & 12 \\ \hline & 1 & 12 \end{array}$$ 8m. — 1m. 12d. = 6m. 18d., Term of credit. $\$40 \div 1.033 = \$38.722$, Ans.

12. $\$350 \div 1.03 = \$339.806$; $\$350 - 339.806 = \$10.194$, Ans.

EQUATION OF PAYMENTS.

2. $$\begin{array}{rcrcr} \$400 & \times & 6 & = & \$2400 \\ 500 & \times & 8 & = & 4000 \\ 1000 & \times & 12 & = & 12000 \\ \hline 1900 & & & &)18400(9\frac{13}{19}\text{m., Ans.} \end{array}$$

3. $$\begin{array}{rcrcr} \$360 & \times & 6 & = & \$2160 \\ 548 & \times & 9 & = & 4932 \\ 775 & \times & 12 & = & 9300 \\ \hline 1683 & & & & 16392 \end{array}$$ $$\begin{array}{r} 16392 \div 1683 = 9.739\text{m.} \\ 30 \\ \hline 22.170\text{d.} \end{array}$$ Ans. 9m. 22d.

4. $\$500 \times 6 = \$3000$; $3000 \div 1000 = 3$m., Ans.

5. $\$600 \times 7 = \$4200$; $\$200 \times 4 = \$800$; $4200 - 800 = 3400$; $600 - (300 + 200) = 100$; $3400 \div 100 = 34$m. $=$ 2yr. 10m., Ans.

EXPLANATION.—$600 for 7m. is the same as $1 for 4200m.; $\frac{1}{3}$ of $600, viz. $200, for 4m. equals $1 for 800m.; 4200m. — 800m. = 3400m.; now if the debtor kept but $1 he might keep it 3400m., but as he keeps $100 ($600 — $\frac{1}{2}$ and $\frac{1}{3}$ of $600

= \$100) he may keep it only $\frac{1}{100}$ of 3400m. = 34m. = 2yr. and 10m. from the beginning.

6. F may keep \$250 for 10m. = \$1 for 2500m.
E may keep \$50 for 4m. = \$1 for 200m.
and \$100 for 8m. = \$1 for 800m.
But E, by making present payment, gives up \$1 for 1000m.;
∴ F may keep \$1 for only 1500m.;
or, \$250 for $\frac{1}{250}$ of 1500m. = 6m., Ans.

7. $\frac{1}{6} \times 2 = \frac{1}{3}$; $\frac{1}{3} \times 3 = 1$; $1 - (\frac{1}{6} + \frac{1}{3}) = \frac{1}{2}$; $\frac{1}{2} \times 6 = 3$; $\frac{1}{3} + 1 + 3 = 4\frac{1}{3}$; $1 - \frac{1}{2} = \frac{1}{2}$; $4\frac{1}{3}$m. $\div \frac{1}{2} = 8\frac{2}{3}$m., Ans.

9.

1st bill due	Mar. 17		
2d "	Apr. 18,	32	days from Mar. 17
3d "	May 4,	48	" "
4th "	May 16,	60	" "
5th "	July 29,	134	" "
6th "	Aug. 1,	137	"

\$75
$50 \times 32 = 1600$
$100 \times 48 = 4800$
$25 \times 60 = 1500$
$200 \times 134 = 26800$
$50 \times 137 = 6850$
5,00)415,50
83d.

83 days from Mar. 17 gives June 8, Ans.

10.

1st bill due	Mar. 31		
2d "	Apr. 15,	15	days from Mar. 31
3d "	Apr. 12,	12	" "
4th "	Apr. 29,	29	" "
5th "	June 14,	75	" "

\$100
$75 \times 15 = 1125$
$200 \times 12 = 2400$
$66 \times 29 = 1914$
$300 \times 75 = 22500$
741 27939

$27939 \div 741 = 38$; 38 days from March 31 gives May 8, Ans.

COMPOUND EQUATION OF PAYMENTS.

3. *Debit.*

1st bill due Feb. 7
2d " Apr. 7, 60 days from Feb. 7
3d " May 7, 90 " "

$$\begin{array}{rcl}
\$800 & & \\
566\frac{2}{3} \times 60 & = & 34000 \\
\underline{433\frac{1}{3}} \times 90 & = & \underline{39000} \\
1800 & & 73000
\end{array}$$

73000 ÷ 1800 = $40\frac{5}{9}$; 41 days from Feb. 7 gives Mar. 19, equated time of debit side.

Credit.

1st bill due Mar. 18
2d " June 26, 100 days from Mar. 18
3d " June 1, 75 " "
4th " June 26, 100 " "

$$\begin{array}{rcl}
\$200 & & \\
1200 \times 100 & = & 120000 \\
300 \times 75 & = & 22500 \\
\underline{800} \times 100 & = & \underline{80000} \\
2500 & & 222500 \\
\underline{1800} & = & \text{Less side of Account.} \\
700 & = & \text{Balance due E.}
\end{array}$$

222500 ÷ 2500 = 89; 89 days from March 18 gives June 15, equated time of credit side.

From Mar. 19 to June 15 = 88 days; 1800 × 88 = 158400; 158400 ÷ 700 = 226, No. of days to be reckoned *forward* from June 15, 1856, which gives Jan. 27, 1857, Ans.

4. From June 4 to Sept. 6 = 94 days; \$800 — \$600 = \$200; 600 × 94 = 56400; 56400 ÷ 200 = 282, No. of days to be reckoned *backward* from June 4, 1857, which gives Aug. 26, 1856, 1st Ans. \$200 × $1.061\frac{2}{3}$ = \$212.333, 2d Ans.

ANALYSIS.

2. $\frac{\$35 \times 8 \times 7}{5} = \$392$, Ans.

3. If 20 is $\frac{5}{9}$, 4 is $\frac{1}{9}$, and $\frac{9}{9} = 4 \times 9 = 36$, **Ans.**

4. $51 \div 17 = 3$; $3 \times 173 = 519$, Ans.

5. $95 \div 19 = 5$; $5 \times 23 = 115$, Ans.

6. 95s. $\div$ 19 = 5s.; 5s. $\times$ 23 = 115s., Ans.

7. $\$74 \div 37 = \$2$; $\$2 \times 50 \times 5 = \$500$, Ans.

8. $64 \div 8 = 8$; $8 \times 9 = 72$; $72 \div 12 = 6$, Ans.

9. $72 \div 9 = 8$; $8 \times 5 = 40$; $40 \div 4 = 10$, Ans.

10. $\$63 \div 9 = \$7$; $\$7 \times 7 = \$49$, Ans.

11. $\frac{2}{5} + \frac{3}{7} = \frac{29}{35}$; $\frac{35}{35} - \frac{29}{35} = \frac{6}{35}$; $6 \div 6 = 1$; 1ft. $\times$ 35 = 35ft., Ans.

12. $12 \times 7 = 84$; $84 \div 24 = 3\frac{1}{2}$, No. months, Ans. Or, Twice the No. of men gives half the time; $\frac{1}{2}$ of 7m. = $3\frac{1}{2}$m., Ans.

13. A builds 5 times 7 rods, ∴ B will build 5 times 9 rods = 45rods in 33 days; Again 44 is $\frac{4}{3}$ of 33, ∴ B, in 44 days, will build $\frac{4}{3}$ of 45 rods = 60 rods, Ans.

14. $\frac{1}{7}$ of 28 = 4, and $\frac{3}{7}$ of 28 = 12; if 12 is $\frac{4}{11}$, then 3 is $\frac{1}{11}$ and 33 is $\frac{11}{11}$; $\frac{1}{5}$ of 55 is 11, and 11 in 33, 3 times; Ans. 3.

15. $\frac{1}{11}$ of 44 = 4, and $\frac{5}{11}$ of 44 = 20; if 20 is $\frac{4}{9}$, then 5 is $\frac{1}{9}$ and 45 is $\frac{9}{9}$; $\frac{1}{3}$ of 15 is 5, and 5 in 45, 9 times; Ans. 9.

16. $\frac{1}{3}$ of 27 = 9, and $\frac{7}{3}$ of 27 = 63; if 63 is $\frac{21}{5}$, then 3 is $\frac{1}{5}$ and 15 is $\frac{5}{5}$; $\frac{1}{12}$ of 60 is 5, and 5 in 15, 3 times; Ans. 3.

17. The hound gains 3 rods in running 27; ∴, as he must gain 13 times 3 rods, he must run 13 times 27 = 351 rods, Ans.

18. He now has 1 *flock*, and if he had one and one half times as many more he would have $2\frac{1}{2}$ ***flocks.*** **Then, not**

having the $2\frac{1}{2}$ sheep, he would have only $97\frac{1}{2}$ *sheep;* $\therefore$ $97\frac{1}{2}$ sheep $= 2\frac{1}{2}$ flocks, and $97\frac{1}{2} \div 2\frac{1}{2} = 39$, No. in 1 flock, Ans.

19. Reasoning as in Ex. 18, $66\frac{2}{3}$ sheep $= 3\frac{1}{3}$ flocks, and $66\frac{2}{3} \div 3\frac{1}{3} = 20$, No. in 1 flock, Ans.

20. $12 \times 7 \times 2000 = 168000$, No. of rations;
$168000 \times 14 = 2352000$, No. ounces, good and bad;
$105 \times 200 \times 16 = \underline{336000}$, No. ounces spoiled;
2016000, No. ounces left to subsist on.
$2016000 \div 168000 = 12$, No. ounces in 1 ration, Ans.

21. Reasoning as in Ex. 20, we find the whole weight to be 2352000oz. $=$ 147000 lb., 1st Ans.
147000 lb. $\div 7 =$ 21000 lb., 2d Ans.

22. As above, whole weight $=$ 147000 lb., 1st Ans.
12oz. $= \frac{6}{7}$ of 14oz.; $\frac{6}{7}$ of 147000 lb. $=$ 126000 lb., 2d Ans.

23. As above, whole weight $=$ 147000 lb., 1st Ans.
If 12oz. is $\frac{6}{7}$, then $\frac{1}{7}$ is 2oz. and $\frac{7}{7} =$ 14oz., 2d Ans.

24. $1 - \frac{1}{4} = \frac{3}{4}$; $\frac{1}{6}$ of $\frac{3}{4} = \frac{1}{8}$; $\frac{1}{4} + \frac{1}{8} = \frac{3}{8}$, portion spent; $1 - \frac{3}{8} = \frac{5}{8}$; if \$5000 $= \frac{5}{8}$, then \$1000 $= \frac{1}{8}$ and $\frac{8}{8} =$ \$8000, Ans.

25. $\dfrac{2 \times 28 \times 60 \times 60}{12 \times 5280} = 3\frac{2}{11}$m., 1st Ans.
$60 \div 3\frac{2}{11} + 1\frac{1}{2} = 20\frac{5}{14}$h., 2d Ans.

26. $\frac{1}{10} + \frac{1}{12} + \frac{1}{15} = \frac{1}{4}$; if $\frac{1}{4}$ in one day, then $\frac{4}{4}$ in 4 days, Ans.

27. $\dfrac{2 \times 57}{75} = 1\frac{13}{25}$ lb., Ans.

28. $129\frac{17}{27} = \frac{3500}{27}$; $\$\frac{11}{7} - \$\frac{7}{5} = \$\frac{6}{35}$, gain per yd.
$\dfrac{200 \times 35 \times 27}{6 \times 3500} = 9$, No. bales, Ans.

EXPLANATION. — Dividing the total gain, \$200, by the gain per yd., $\$\frac{6}{35}$, gives the No. of yards; then dividing the No. of yards by the No. of yards in a bale, $\frac{3500}{27}$, gives the No. of bales.

29. $246\frac{18}{133} = \frac{32736}{133}$; $\$\frac{5}{12} - \$\frac{3}{11} = \$\frac{19}{132}$, gain per lb.;
$\frac{248 \times 132 \times 133}{19 \times 32736} = 7$, No. bales, Ans.

30. Since the miller took $\frac{1}{16}$ for grinding, 60bbl. of flour were made out of $\frac{15}{16}$ of the wheat; ∴ enough wheat was carried to mill to make 64bbl. of flour.

$4\frac{1}{2}$bush. × 64 = 288bush. carried to mill.
$3\frac{3}{4}\frac{1}{8}$c. × 288 = \$10.50 for carting wheat.
25c. × 60 = 15.00 for barrels.
$\$1\frac{1}{4}$ × 60 = 75.00 for carrying flour.
25c. × 60 = 15.00 for agent.
\$115.50, total expense.

\$367.50 — \$115.50 = \$252, net receipts.
\$252 ÷ 288 = $87\frac{1}{2}$c., Ans.

PROPORTION.

3. $\cancel{4}$ft. : $\overset{60}{\cancel{2}\cancel{4}\cancel{0}}$ft. :: 3ft. : 180ft., Ans.

4. $\cancel{3}$ft. : $\overset{60}{\cancel{1}\cancel{8}\cancel{0}}$ft. :: 4ft. : 240ft., Ans.

5. $\overset{6}{\cancel{2}\cancel{4}\cancel{0}}$ft. : $\cancel{4}$ft. :: $18\cancel{0}$ft. : 3ft., Ans.

6. $\overset{6}{\cancel{1}\cancel{8}\cancel{0}}$ft. : $\cancel{3}$ft. :: $24\cancel{0}$ft. : 4ft., Ans.

8. 11 men : 33 men :: 67d. : 201d., Ans.

9. 3yr. : 11yr. :: \$2700 : \$9900, Ans.

10. \$9900 : \$2700 :: 11yr. : 3yr., Ans.

14. 913£ 6s. 6d. : 130£ 9s. 6d. :: 94a. 3r. 2rd. : 4th term.
913.325£ : 130.475£ :: 94.7625a. : 13.5375a.
13.5375a. = 13a. 2r. 6rd., Ans.

15. 130£ 9s. 6d. : 913£ 6s. 6d. :: 13a. 2r. 6rd. : 4th term.
130.475£ : 913.325£ :: 13.5375a. : 94.7625a.
94.7625a. = 94a. 3r. 2rd., Ans.

16. 94a. 3r. 2rd. : 13a. 2r. 6rd. :: 913£ 6s. 6d. : 4th term;
94.7625a. : 13.5375a. :: 913.325£ : 130.475£;
130.475£ = 130£ 9s. 6d., Ans.

17. $1\frac{3}{8}$yd. : $\frac{3}{4}$yd. :: $12\frac{1}{2}$yd. : $6\frac{9}{11}$yd., Ans.

18. $\frac{7}{16}$: $\frac{9}{16}$, or 7 : 9 :: $1163 : $$1495\frac{2}{7}$, Ans.

19. 10 men : 6 men :: 40d. : 24d., Ans.

20. $\frac{3}{7}$bbl. : 14bbl. :: $3.60 : $117.60, Ans.

21. $\frac{4}{9}$a. : $15\frac{3}{16}$a. :: $36.40 : $1242.15, Ans.

22. 4h. : 6h. :: 2d. : 3d., Ans.

23. 3c. : 45c. :: 2bbl. : 30bbl., Ans.

24. 4d. : 16d. :: 4 men : 16 men.
16 men — 4 men = 12 men, Ans.

25. 45d. : 30d. :: 15 men : 10 men.
15 men — 10 men = 5 men, Ans.

26. $25000 : $500 :: $15000 : $300, Ans.

27. $\frac{2}{3}$ of $\frac{3}{5}$ or $\frac{2}{5}$: 1 :: $20000 : $50000, Ans.

28. $450 : $300 :: 9m. : 6m., Ans.

29. $8 : $12 :: 4oz. : 6oz., Ans.

30. 11 + 19 = 30 : 11 :: $45.90 : $16.83, A's share.
30 : 19 :: $45.90 : $29.07, B's share.

31. 5 days : 45 days :: 13 men : 117 men; 117 men — 13 men = 104 men, Ans.

32. $\frac{3}{4}$yd. : $\frac{5}{4}$yd. :: 63yd. : 105yd., Ans.

33. 10 horses : 14 horses :: 35bush. : 49bush., Ans.

34. 1yr. : $3\frac{1}{2}$yr. :: $42 : $147, Ans.

35. $(13\frac{1}{2} + 12) \times 2 = 51$; 51ft. × 9 = 459ft., area of the walls;
2ft. × 3 = 6ft., area of a yard of paper;
6ft. : 459ft. :: 1yd. : $76\frac{1}{2}$yd., Ans.

36. $2 : $168 :: 1gal. : 84gal.; 84gal. — 63gal. = 21gal., Ans.

37. 30d. : 97d. :: 30 men : 97 men, Ans.

38. 1800 men : 1500 men :: 12m. : 10m., Ans.

39. 2rd. : 8rd. :: 20rd. : 80rd., Ans.

40. 366rev. : 1rev. :: 365d. : $\frac{365}{366}$d. = 23h. 56$\frac{4}{61}$m., Ans.

41. 8ft. : 16ft. :: 6 men : 12 men, Ans.

42. 4d. : 12d. :: 12 men : 36 men, Ans.

43. 7lb. : 19lb. :: $3.50 : $9.50, Ans.

44. 8h. : 12h. :: 6d. : 9d., Ans.

45. $300 : $500 :: 3m. : 5m., Ans.

46. $\frac{7}{8}$yd. : 1$\frac{1}{2}$yd. :: 9yd. : 15$\frac{3}{7}$yd., Ans.

47. 45m. : 360m. :: 1 pipe : 8 pipes, Ans.

48. $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{47}{60}$; $\frac{47}{60} : \frac{60}{60}$ :: 1h. : 1$\frac{13}{47}$h., Ans.

49. $\frac{1}{6} - \frac{1}{8} = \frac{1}{24}$; $\frac{1}{24} : \frac{24}{24}$ :: 1d. : 24d., Ans.

50. 16t. : 72t. :: 2 horses : 9 horses, Ans.

51. $25.50 : $1925.25 :: 1a. : 75.5a. = 75a. 2r., Ans.

52. pr. of horse : pr. of oxen :: $75 : 4th term.
$5^2 : 7^2$, or 25 : 49 :: $75 : $147, Ans.

53. 400 men : 300 men :: 60d. : 45d., Ans.

54. 12oz. : 14oz. :: 120d. : 140d., Ans.

55. $\frac{5}{11} : \frac{11}{16} :: \$25000 : \frac{\$25000 \times 11 \times 11}{16 \times 5} =$
$37812.50, Ans.

56. 100lb. : 37$\frac{1}{2}$lb. :: $27 : 10.12\frac{1}{2}$, Ans.

57. .25 : .376 :: $750 : $1128, Ans.

58. $4^3 : 3^3$ :: B's : A's; 64 : 27 :: $12800 : $5400, Ans.

59. 1sec. : 207sec. :: 19m. : 3933m., Ans.

COMPOUND PROPORTION.

4. $$\left.\begin{array}{lcl}8\text{ft.} & : & 24\text{ft.}\\ 6\text{d.} & : & 12\text{d.}\end{array}\right\} :: 6\text{ men} : 36\text{ men, Ans.}$$

5. $$\left.\begin{array}{rcr}24\text{d.} & : & 16\text{d.}\\ 9\text{h.} & : & 12\text{h.}\\ 30\text{ft.} & : & 45\text{ft.}\\ 8\text{ft.} & : & 9\text{ft.}\\ 3\text{ft.} & : & 6\text{ft.}\end{array}\right\} :: 3\text{ men} : 9\text{ men, Ans.}$$

BY CANCELING.

$$\frac{\cancel{16} \times \cancel{12} \times \overset{9}{\cancel{45}} \times \cancel{9} \times \cancel{6} \times \cancel{3}}{\underset{\cancel{2}}{\cancel{24}} \times \cancel{9} \times \underset{\cancel{5}}{\cancel{30}} \times \cancel{8} \times \cancel{3}} = 9,\text{ Ans.}$$

6. $$\left.\begin{array}{lcl}6\text{ persons} & : & 10\text{ persons}\\ 8\text{m.} & : & 14\text{m.}\end{array}\right\} :: \$600 : \$1750,\text{ Ans.}$$

7. $$\left.\begin{array}{lcr}9\text{hhd.} & : & 50\text{ tierces}\\ 12\text{cwt.} & : & 2\tfrac{1}{2}\text{cwt.}\\ 60\text{m.} & : & 300\text{m.}\end{array}\right\} :: \$50 : \$289.35\tfrac{5}{27},\text{ Ans.}$$

8. $$\left.\begin{array}{lcl}\$100 & : & \$300\\ 12\text{m.} & : & 9\text{m.}\end{array}\right\} :: \$8 : \$18,\text{ Ans.}$$

9. $$\left.\begin{array}{lcl}\$300 & : & \$100\\ 9\text{m.} & : & 12\text{m.}\end{array}\right\} :: \$18 : \$8,\text{ Ans.}$$

10. $$\left.\begin{array}{rcr}\$300 & : & \$100\\ \$8 & : & \$18\end{array}\right\} :: 12\text{m.} : 9\text{m., Ans.}$$

11. $$\left.\begin{array}{lcl}\$8 & : & \$18\\ 9\text{m.} & : & 12\text{m.}\end{array}\right\} :: \$100 : \$300,\text{ Ans.}$$

12. $$\left.\begin{array}{rcr}\$300 & : & \$100\\ \$6 & : & \$18\\ 9\text{m.} & : & 12\text{m.}\end{array}\right\} :: 6\text{ per cent.} : 8\text{ per cent., Ans.}$$

EXPLANATION. — 6 per cent. is assumed, it being a customary rate. Any other rate may be assumed just as well. $300 require a smaller rate than $100 to gain a given interest. It requires a larger rate to gain \$18 than to gain \$6, the interest of \$100 for 1 year at the assumed rate. It requires a larger rate to gain a given sum in 9 months than in 12 months.

13.
$$\left.\begin{array}{rcl} 2\text{d.} & : & 40\text{d.} \\ 162\text{d.} & : & 81\text{d.} \end{array}\right\} :: 8\text{oz.} : 80\text{oz.} = 5\text{lb., Ans.}$$

14.
$$\left.\begin{array}{lcl} 25\text{in. long} & : & 21\tfrac{13}{16}\text{in. long} \\ 6\text{in. wide} & : & 8\text{in. wide} \\ 3\text{in. thick} & : & 4\text{in. thick} \\ 10.47 & : & 19.26 \\ 2 & : & 33 \end{array}\right\} :: \$2725 : \$128293, \text{ Ans.}$$

$$21\tfrac{13}{16} = \tfrac{349}{16}.$$

ARRANGEMENT FOR CANCELING.

$$\frac{349 \times 8 \times 4 \times 1926 \times 33 \times 2725\$}{16 \times 25 \times 6 \times 3 \times 1047 \times 2} = \$128293, \text{ Ans.}$$

15.
$$\left.\begin{array}{lcl} 22\text{d.} & : & 5\text{d.} \\ 540\text{m.} & : & 726\text{m.} \\ 9\text{deg.} & : & 2\text{deg.} \\ 465\text{ft. long} & : & 168\tfrac{3}{4}\text{ft. long} \\ 3\tfrac{2}{3}\text{ft. wide} & : & 7\tfrac{1}{2}\text{ft. wide} \\ 4\tfrac{2}{3}\text{ft. deep} & : & 2\tfrac{4}{5}\text{ft. deep} \end{array}\right\} :: 496\text{ men} : 15\text{ men, Ans.}$$

ARRANGEMENT FOR CANCELING.

$$\frac{5 \times 726 \times 2 \times 168\tfrac{3}{4} \times 7\tfrac{1}{2} \times 2\tfrac{4}{5} \times 496}{22 \times 540 \times 9 \times 465 \times 3\tfrac{2}{3} \times 4\tfrac{2}{3}} \text{ or, reducing,}$$

$$\frac{5 \times 726 \times 2 \times 675 \times 15 \times 3 \times 14 \times 3 \times 496\text{ men}}{22 \times 540 \times 9 \times 4 \times 465 \times 2 \times 11 \times 5 \times 14}$$

$$= 15\text{ men, Ans.}$$

Explanation.—It requires less men to dig a trench in 22 days than in 5; more to do it when the days are 540 minutes (9h.) long than when they are 726 minutes (12h. 6m.); less when it is of 2 degrees of hardness than when it is of 9; less when it is 168¾ft. long than when it is 465ft. long; more when it is 7½ft. wide than when it is 3⅔ft. wide; and less when it is 2⅘ft. deep than when it is 4⅔ft. deep.

CONJOINED PROPORTION.

4. 2 $\cancel{5}\cancel{6}$£ Ga. = ——£ S.?
3 $\cancel{4}\cancel{5}$£ N. J. = $\cancel{2}\cancel{8}$£ Ga.
$\cancel{2}$ $\cancel{1}\cancel{6}$£ N. Y. = $\cancel{1}\cancel{5}$£ N. J.
$\cancel{6}$£ N. E. = $\cancel{8}$£ N. Y.
9£ S. = $\cancel{1}\cancel{2}$£ N. E. $\cancel{2}$
54£ S., Ans.

5. 5 $\cancel{2}\cancel{0}$ $\cancel{1}\cancel{8}\cancel{0}\cancel{0}$a. Br. = ——a. A.?
11 $\cancel{3}\cancel{3}$a. L. = $\cancel{4}\cancel{0}$a. Br.
25a. M. = $\cancel{2}\cancel{0}$a. L.
$\cancel{5}\cancel{0}$a. Bo. = $\cancel{4}\cancel{5}$a. M. $\cancel{9}$.
$\cancel{2}\cancel{0}$a. A. = $\cancel{3}\cancel{0}$a. Bo.
1375a. A., Ans.

6. 27w. = ——b.?
12m. = 18w.
20b. = 15m.
6480 ÷ 270 = 24b., Ans.

7. $\cancel{5}\cancel{0}$ S. d. = —— A. e.?
$\cancel{2}$ $\cancel{8}\cancel{8}$ G. C. = $\cancel{1}\cancel{9}\cancel{0}$ S. d. $\cancel{2}$
$\cancel{9}\cancel{5}$ I. s. = $\cancel{4}\cancel{4}$ G. C.
11 $\cancel{2}\cancel{2}$ L. d. = $\cancel{4}\cancel{5}$ I. s. $\cancel{1}\cancel{5}$
$\cancel{1}\cancel{5}\cancel{0}$ E. s. = $\cancel{1}\cancel{6}\cancel{1}$ L. d.
$\cancel{3}$ $\cancel{4}\cancel{8}\cancel{3}$ A. e. = $\cancel{1}\cancel{0}\cancel{0}\cancel{0}$ E. s.
11 A. e., Ans.

PROFIT AND LOSS.

(ART. **266**, p. 178.)

1. \$7 × 16 = \$112, sum received;
\$100, sum paid;
16) \$12, total gain; } Ans.
\$ .75, gain per bbl., }

2. $\$4 \times 75 = \$300$, sum received;
$\$250$, sum paid;
$$\left.\begin{array}{ll} 75)\$50, & \text{total gain;} \\ \$\ .66\frac{2}{3}, & \text{gain per yd.,} \end{array}\right\} \text{Ans.}$$

3. 13cwt. 1qr. 19lb. = 1344lb.; $107.52, sum paid;
$6\frac{1}{4}$c. × 1344 = $84. , sum received;
$$\left.\begin{array}{ll} 1344)\$23.52, & \text{total loss,} \\ .01\frac{3}{4}, & \text{loss per lb.} \end{array}\right\} \text{Ans.}$$

4. $164 \times \frac{7}{4} = 287$, No. yd. cassimere;
$\$3 \times \frac{3}{4} = \$2.25$, price of cassimere;
$\$3 \times 164 = \$492$, sum received for broadcloth;
$\$2.25 \times 287 = \$645.75$, sum received for cassimere;
$1137.75, total sum received;
$1107 , sum paid;
$30.75, sum gained, Ans.

(Art. **267**, p. 179.)

3. $\$3670.80 - \$3496 = \$174.80$, total gain;

$3496 : $174.80 :: 100 per cent. : 5 per cent., Ans. by Rule 1.

$\frac{174.80}{3496} = .05$, Ans. by Rule 2.

4. Int. on $\$3709.75\frac{475}{1979}$ for 60d. = $\$37.0975\frac{475}{1979}$
Int. on the same for 3d. = $1.8548\frac{1598}{1979}$
Int. on the same for 63d. = $\$38.9524\frac{4}{1979} = \$38.95\frac{475}{1979}$;

$\$3709.75\frac{475}{1979} - \$38.95\frac{475}{1979} = \$3670.80$, sum received at bank.

$\$3670.80 - \$3496 = \$174.80$, sum gained.

$3496 : $174.80 :: 100 per cent. : 5 per cent., Ans.

5. $4 : $1 :: 100 per cent. : 25 per cent., Ans.

6. $5 : $1 :: 100 per cent. : 20 per cent., Ans.

7. $4000 × .06 = $240, Int. on $4000 for 1yr.;
190, cost of storage;
4000, 1st cost;
$4430, total cost;
$4310, sum received;
$120, total loss;

$4000 : $120 :: 100 per cent. : 3 per cent., Ans.

(ART. **268**, p. 180.)

3. 12½c. × .10 = 1¼c.; 12½c, — 1¼c. = 11¼c., Ans. by Rule 1.
100 per cent. : 90 per cent. :: 12½c. : 11¼c., Ans. by Rule 2.

4. 12½c. × .20 = 2½c.; 12½c. + 2½c. = 15c., Ans.

5. $600 × .06½ = $39; $600 + $39 = $639 for the 6 shares, or $639 ÷ 6 = $106.50 for 1 share, Ans.

(ART. **269**, p. 181.)

3. $30 ÷ 6 = $5, sum received per yard;
112 per cent. : 100 per cent. :: $5 : 4.46\frac{3}{7}$, Ans.

4. $800 ÷ 10 = $80, sum received per share;
80 per cent. : 100 per cent. :: $80 : $100, Ans.

(ART. **270**, p. 182.)

4. $5000 : $3000 :: 125 per cent. : 75 per cent.;
100 per cent. — 75 per cent. = 25 per cent. loss, Ans.

5. $175 : $200 :: 105 per cent. : 120 per cent.;
120 per cent. — 100 per cent. = 20 per cent., Ans.

6. $4000 : $3000 :: 120 per cent. : 90 per cent.;
100 per cent. — 90 per cent. = 10 per cent. loss, Ans.

7. $5000 : $6000 :: 125 : 150;
150 — 100 = 50 per cent., Ans.

(ART. **271**, p. 183.)

2. 90 : 100 :: \$18 : \$20, Ans.

(ART. **271**, p. 184.)

3. 90 : 125 :: \$4 : $\$5.55\frac{5}{9}$, Ans.

4. 88 : 110 :: \$8 : \$10, Ans.

5. 95 : 105 :: \$150 : $\$165.78\frac{18}{19}$, Ans.

6. 80 : 95 :: \$175 : $\$207.81\frac{1}{4}$, Ans.

(ART. **272**, p. 184.)

1. \$15 × 5 = \$75
   \$17 × 5 = \$85
   Sum received, \$160
   Cost, \$155
   Ans., Gain, \$5

2. \$4.50, selling price.
   \$3.00, cost.
   \$1.50, gain per hat.
   \$3 : \$1.50 :: 100 : 50, Ans.

3. \$1.50, cost;
   \$1.25, sum received;
   25, loss.
   \$1.50 : 25c. :: 100 : $16\frac{2}{3}$, Ans.

4. 90 : 125 :: 75c. : $\$1.04\frac{1}{6}$, Ans.

5. 75 : 150 :: \$3 : \$6, Ans.

6. \$4 : \$4.25 :: 90 : $95\frac{5}{8}$; 100 — $95\frac{5}{8}$ = $4\frac{3}{8}$, loss, Ans.

7. 88 : 100 :: \$40 : $\$45.45\frac{5}{11}$, Ans.

8. $$\frac{\$350 \times 8 \times 100 \times 100 \times 105}{100 \times 15 \times 95 \times 90 \times 100} = \$2.29\tfrac{41}{171}, \text{ Ans.}$$

EXPLANATION.—\$3.50 = $\frac{\$350}{100}$; divide by $1\frac{7}{8}$, i.e. multiply by $\frac{8}{15}$ to find the cost per square yard; because the cloth shrinks 5 per cent. in width 100yd. becomes 95yd., ∴ the cost of 1yd. multiplied by 100 and divided by 95 gives the cost of 1yd. after it is shrunk in width; for a similar reason multiply by $\frac{100}{90}$, because of the shrinkage in length; finally multiply by $\frac{105}{100}$ because it is to be sold so as to *gain* 5 per cent. Then cancelling and reducing, the result is $\$2.29\frac{41}{171}$.

PARTNERSHIP.

2. $\$3000$, A's stock; $\frac{3000}{15000} = \frac{1}{5}$, A's share of stock.
5000, B's stock; $\frac{5000}{15000} = \frac{1}{3}$, B's share of stock.
7000, C's stock; $\frac{7000}{15000} = \frac{7}{15}$, C's share of stock.
$\$15000$, Total stock.

$$\left.\begin{array}{l}\$3000 \times \frac{1}{5} = \$600, \text{ A's gain}\\ \$3000 \times \frac{1}{3} = \$1000, \text{ B's gain}\\ \$3000 \times \frac{7}{15} = \$1400, \text{ C's gain}\end{array}\right\}\text{Ans. by Rule 1.}$$

$$\left.\begin{array}{l}\$15000 : \$3000, \text{ or } \$15 : \$3 :: \$3000 : \$600, \text{ A's gain}\\ \$15000 : \$5000, \text{ or } \$15 : \$5 :: \$3000 : \$1000, \text{ B's gain}\\ \$15000 : \$7000, \text{ or } \$15 : \$7 :: \$3000 : \$1400, \text{ C's gain}\end{array}\right\}$$

Ans. by Rule 2.

3. $\frac{1}{5}$ of $\$600 = \$120$, A's; $\frac{1}{3}$ of $\$600 = \$200$, B's; $\frac{7}{15}$ of $\$600 = \$280$, C's, Ans.

4. $\frac{1}{3} + \frac{2}{7} = \frac{13}{21}$; $1 - \frac{13}{21} = \frac{8}{21}$, C's share of capital.

$$\left.\begin{array}{l}\frac{1}{3} \text{ of } \$4284.21 = \$1428.07, \text{ A's gain}\\ \frac{2}{7} \text{ of } \$4284.21 = \$1224.06, \text{ B's gain}\\ \frac{8}{21} \text{ of } \$4284.21 = \$1632.08, \text{ C's gain}\end{array}\right\}\text{Ans.}$$

5. $\$2000 + \$1500 + \$1000 = \$4500$, sum of debts.

$$\left.\begin{array}{l}\frac{2000}{4500} \text{ or } \frac{4}{9} \text{ of } \$3000 = \$1333.33\frac{1}{3}, \text{ A's share}\\ \frac{1500}{4500} \text{ or } \frac{1}{3} \text{ of } \$3000 = \$1000, \text{ B's share}\\ \frac{1000}{4500} \text{ or } \frac{2}{9} \text{ of } \$3000 = \$666.66\frac{2}{3}, \text{ C's share}\end{array}\right\}\text{Ans.}$$

6. $10 + 8 + 12 = 30$;

$$\left.\begin{array}{l}\frac{10}{30} \text{ or } \frac{1}{3} \text{ of } \$150 = \$50, \text{ A's share}\\ \frac{8}{30} \text{ or } \frac{4}{15} \text{ of } \$150 = \$40, \text{ B's share}\\ \frac{12}{30} \text{ or } \frac{2}{5} \text{ of } \$150 = \$60, \text{ C's share}\end{array}\right\}\text{Ans.}$$

7. $7\frac{3}{7} + 22\frac{4}{7} = 30$;

$$\left.\begin{array}{l}30 : 7\frac{3}{7} :: \$15 : \$3.71\frac{3}{7}, \text{ A's share}\\ 30 : 22\frac{4}{7} :: \$15 : \$11.28\frac{4}{7}, \text{ B's share}\end{array}\right\}\text{Ans.}$$

8. $\$12000 + \$9000 + \$14000 + \$15000 = \$50000$, total cargo.

$$\left.\begin{array}{l} 50 : 12 :: \$25000 : \$6000, \text{ A's gain} \\ 50 : 9 :: \$25000 : \$4500, \text{ B's gain} \\ 50 : 14 :: \$25000 : \$7000, \text{ C's gain} \\ 50 : 15 :: \$25000 : \$7500, \text{ D's gain} \end{array}\right\} \text{Ans.}$$

9. $2 + 5 + 9 = 16$, total No. of equal parts.

$$\left.\begin{array}{l} 16 : 2 :: \$800 : \$100, \text{ A's part} \\ 16 : 5 :: \$800 : \$250, \text{ B's part} \\ 16 : 9 :: \$800 : \$450, \text{ C's part} \end{array}\right\} \text{Ans.}$$

10. 1 cow = 6 sheep, ∴ 5 cows or 3 oxen = 30 sheep, and ∴ 1 ox = 10 sheep; 5 oxen or 4 horses = 50 sheep, and ∴ 1 horse = $12\frac{1}{2}$ sheep.

A's stock.	B's stock.	C's stock.	Total stock.
3h. = $37\frac{1}{2}$s.	2h. = 25s.	4h. = 50s.	
6ox. = 60 s.	3ox. = 30s.	5ox. = 50s.	
8c. = 48 s.	2c. = 12s.	4c. = 24s.	
10 s.	40s.	50s.	
$155\frac{1}{2}$s. +	107s. +	174s. =	$436\frac{1}{2}$s.

$$\left.\begin{array}{l} \$523.80 \times \dfrac{155\frac{1}{2}}{436\frac{1}{2}} = \$186.60, \text{ A's share} \\ \$523.80 \times \dfrac{107}{436\frac{1}{2}} = \$128.40, \text{ B's share} \\ \$523.80 \times \dfrac{174}{436\frac{1}{2}} = \$208.80, \text{ C's share} \end{array}\right\} \text{Ans.}$$

11. The ratio of $\frac{1}{3}$ to $\frac{1}{2} = 2 : 3$ (Art. 246); $2 + 3 = 5$, sum of shares.

$$\left.\begin{array}{l} \frac{2}{5} \text{ of } \$1000 = \$400, \text{ 1st son's share} \\ \frac{3}{5} \text{ of } \$1000 = \$600, \text{ 2d son's share} \end{array}\right\} \text{Ans.}$$

12. $\frac{2}{3}$, $\frac{5}{4}$, and $\frac{5}{2}$ are to each other as $\frac{8}{12}$, $\frac{15}{12}$, and $\frac{30}{12}$, or as 8, 15, and 30; $8 + 15 + 30 = 53$, sum of shares.

$$\left.\begin{array}{l} \frac{8}{53} \text{ of } \$500 = \$75.47\frac{9}{53}, \text{ A's share} \\ \frac{15}{53} \text{ of } \$500 = \$141.50\frac{50}{53}, \text{ B's share} \\ \frac{30}{53} \text{ of } \$500 = \$283.01\frac{47}{53}, \text{ C's share} \end{array}\right\} \text{Ans.}$$

13. \$2000 + \$1500 + \$1000 = \$4500, sum bequeathed.

$$\left.\begin{array}{l}\frac{2000}{4500}=\frac{4}{9};\ \frac{4}{9}\text{ of }\$2700=\$1200\text{, 1st son's share}\\ \frac{1500}{4500}=\frac{1}{3};\ \frac{1}{3}\text{ of }\$2700=\$\ \ 900\text{, 2d son's share}\\ \frac{1000}{4500}=\frac{2}{9};\ \frac{2}{9}\text{ of }\$2700=\$\ \ 600\text{, daughter's share}\end{array}\right\}\text{Ans.}$$

14. A furnishes $\frac{2}{3}$ and B $\frac{1}{3}$ of the capital, ∴ A must have $\frac{2}{3}$ and B $\frac{1}{3}$ of the profits, 1st Ans.
\$1000 ÷ 3 = \$333$\frac{1}{3}$, B's capital; \$333$\frac{1}{3}$ ÷ 100 = \$3.33$\frac{1}{3}$, 2d Ans.

COMPOUND PARTNERSHIP.

2. \$1200 × 8 = \$9600, A's stock.
\$1700 × 11 = \$18700, B's stock.
\$28300, Total stock.

$$\left.\begin{array}{l}283 : \ \ 96 :: \$500 : \$169.61\tfrac{37}{283}\text{, A's loss}\\ 283 : 187 :: \$500 : \$330.38\tfrac{246}{283}\text{, B's loss}\end{array}\right\}\text{Ans.}$$

3. A's stock.
\$3000 × 2 = \$ 6000
4000 × 6 = 24000
2000 × 4 = 8000
\$38000

B's stock.
\$3000 × 5 = \$15000
2500 × 3 = 7500
4000 × 4 = 16000
\$38500

C's stock.
\$3000 × 5 = \$15000
3500 × 3 = 10500
3000 × 4 = 12000
\$37500

\$38000 + \$38500 + \$37500 = \$114000, sum of products.

$$\left.\begin{array}{l}1140 : 380 :: \$4000 : \$1333.33\tfrac{1}{3}\text{, A's share}\\ 1140 : 385 :: \$4000 : \$1350.87\tfrac{41}{57}\text{, B's share}\\ 1140 : 375 :: \$4000 : \$1315.78\tfrac{54}{57}\text{, C's share}\end{array}\right\}\text{Ans.}$$

Explanation.—A has \$3000 in till Mar. 1, 2 months; then he has \$4000 in till Sept. 1, 6 months; and, finally, he has \$2000 in for the 4 remaining months of the year; ∴ \$3000 × 2 + \$4000 × 6 + \$2000 × 4 = \$38000 may be considered A's product. In a similar manner the products of B and C are found.

4.

$$\begin{array}{rcl} & \text{A's stock.} & \\ \$1000 \times 3 &=& \$3000 \\ 1500 \times 4 &=& 6000 \\ 700 \times 5 &=& \underline{3500} \\ && \$12500 \end{array} \qquad \begin{array}{rcl} & \text{B's stock.} & \\ \$1200 \times 3 &=& \$3600 \\ 1500 \times 4 &=& 6000 \\ 2700 \times 5 &=& \underline{13500} \\ && \$23100 \end{array}$$

$$\begin{array}{rcl} & \text{C's stock} & \\ \$1800 \times 3 &=& \$5400 \\ 1400 \times 4 &=& 5600 \\ 900 \times 5 &=& \underline{4500} \\ && \$15500 \end{array}$$

$12500 + $23100 + $15500 = $51100, sum of products.
$1000 + $1200 + $1800 = $4000, stock in at first;
$1500 + $1500 + $1400 = $4400, greatest stock in trade;
$700 + $2700 + $900 = $4300, stock in finally;
$4400 × .10 = $440, total gain, 1st Ans.

$$\left.\begin{array}{l} \frac{125}{511} \text{ of } \$440 = \$107.63\frac{107}{511}, \text{ A's share} \\ \frac{231}{511} \text{ of } \$440 = \$198.90\frac{210}{511}, \text{ B's share} \\ \frac{155}{511} \text{ of } \$440 = \$133.46\frac{104}{511}, \text{ C's share} \end{array}\right\} \text{2d and 3d Ans.}$$

5. A's stock,
10 oxen × 20 = 200
15 cows or 10 oxen × 14 = 140
99 sheep or 9 cows or 6 oxen × 26 = 156
A's stock equals 1 ox for 496 weeks.

B's stock,
7 oxen × 24 = 168
12 cows or 8 oxen × 20 = 160
66 sheep or 6 cows or 4 oxen × 25 = 100
B's stock equals 1 ox for 428 weeks.

C's stock,
25 oxen × 8 = 200
12 cows or 8 oxen × 12 = 96
33 sheep or 3 cows or 2 oxen × 15 = 30
C's stock equals 1 ox for 326 weeks.

496 + 428 + 326 = 1250 oxen for 1 week, total stock.
$300 ÷ 1250 = 24c., cost per week for 1 ox.
24c. × 2 = 48c.; 48c. ÷ 3 = 16c., cost per wk. for 1 cow.
16c. ÷ 11 = $1\frac{5}{11}$c., cost per week for 1 sheep.

$1\frac{5}{11}$c. × 99 × 26 = $37.44, what A pays for sheep;
16c. × 15 × 14 = 33.60, what A pays for cows;
24c. × 10 × 20 = 48.00, what A pays for oxen;
$\frac{496}{1250}$ of $300 = $119.04, A's share of expense.

$1\frac{5}{11}$c. × 66 × 25 = $24.00, what B pays for sheep;
16c. × 12 × 20 = 38.40, what B pays for cows;
24c. × 7 × 24 = 40.32, what B pays for oxen;
$\frac{428}{1250}$ of $300 = $102.72, B's share of expense.

$1\frac{5}{11}$c. × 33 × 15 = $ 7.20, what C pays for sheep;
16c. × 12 × 12 = 23.04, what C pays for cows;
24c. × 25 × 8 = 48.00, what C pays for oxen;
$\frac{326}{1250}$ of $300 = $78.24, C's share of expense.

TAXES.

2. $1.40 × 428 = $599.20, sum assessed on the polls.
$3599.20 — $599.20 = $3000, sum assessed on property.
$350000 + $250000 = $600000, am't of taxable property.
$3000 ÷ 600000 = 5 mills, tax on $1.
$4000 + $6500 = $10500, E's taxable property.

$10500 × .005 = $52.50, tax on E's property.
$1.40 × 3 = 4.20, E's tax on polls.
$56.70, E's entire tax, Ans.

3. $5000 × .005 + $1.40 × 2 = $25 + $2.80 = $27.80, A's tax.
$2400 ÷ .005 + $1.40 × 3 = $12 + $4.20 = $16.20, B's tax.
$3600 × .005 + $1.40 = $18 + $1.40 = $19.40, C's tax.
$1690 × .005 + $1.40 × 2 = $8.45 + $2.80 = $11.25, D's tax.

4. **A's tax.**

On $10000 = $500
On polls, $1.50 × 3 = 4.50
Total = $504.50

B's tax.

On $5000 = $250
On polls, $1.50 × 2 = 3
Total = $253

C's tax.

On $7000 = $350
On $500 = 25
On polls, $1.50 × 2 = 3
Total = $378

D's tax.

On $6000 = $300
On 1 poll = 1.50
Total = $301.50

E's tax.

On $10000 = $500
On $4000 = 200
On $700 = 35
On $5 = 0.25
On polls, $1.50 × 2 = 3
Total = $738.25

F's tax.

On $5000 = $250
On $900 = 45
On $10 = 0.50
On $5 = 0.25
Total = $295.75

G's tax.

On $5000 = $250

H's tax.

On $3000 = $150
On $500 = 25
On $30 = 1.50
On polls, $1.50 × 2 = 3
Total = $179.50

J's tax.

On $3000 = $150
On $30 = 1.50
On $5 = 0.25
On polls, $1.50 × 3 = 4.50
Total = $156.25

5. $1.40 × 385 = $539, sum assessed on the polls.
$3184 — $539 = $2645, sum assessed on the property.
$2645 ÷ 1058000 = 2½ mills, tax on $1 = ¼ of 1 per cent.
$33333⅓ × .002½ + $1.40 × 2 = $83.33⅓ + $2.80 = $86.13⅓, A's tax.
$ 6666⅔ × .002½ + $1.40 × 3 = $16.66⅔ + $4.20 = $20.86⅔, B's tax.
$16666⅔ × .002½ + $1.40 = $41.66⅔ + $1.40 = $43.06⅔, C's tax.
$50000 × .002½ + $1.40 × 2 = $125 + $2.80 = $127.80, D's tax.

$\$25000 \times .002\frac{1}{2} + \$1.40 \times 4 = \$62.50 + \$5.60 =$ $\$68.10$, E's tax.

$\$75000 \times .002\frac{1}{2} + \$1.40 \times 2 = \$187.50 + \$2.80 =$ $\$190.30$, F's tax.

$\$15533\frac{1}{3} \times .002\frac{1}{2} + \$1.40 = \$38.83\frac{1}{3} + \$1.40 =$ $\$40.23\frac{1}{3}$, G's tax.

ALLIGATION.

(ART. **285**, p. 194.)

2.
$$\begin{array}{rcrcr}
\$1.05 & \times & 75 & = & \$78.75 \\
1.20 & \times & 25 & = & 30.00 \\
1.50 & \times & 5 & = & 7.50 \\
2.00 & \times & \underline{20} & = & \underline{40.00} \\
 & & 125 &) & \underline{\$156.25} \\
 & & \text{Ans.} & & \$1.25
\end{array}$$

3.
$$\begin{array}{rcrcr}
4c. & \times & 5 & = & 20c. \\
6c. & \times & 4 & = & 24c. \\
9c. & \times & 2 & = & 18c. \\
11c. & \times & 2 & = & 22c. \\
13c. & \times & \underline{4} & = & \underline{52c.} \\
 & & 17 &) & \underline{136c.} \\
 & & \text{Ans.} & & 8c.
\end{array}$$

4.
$$\begin{array}{rcrcr}
12c. & \times & 23 & = & 276c. \\
20c. & \times & 7 & = & 140c. \\
25c. & \times & 5 & = & 125c. \\
38c. & \times & 8 & = & 304c. \\
40c. & \times & 13 & = & 520c. \\
56c. & \times & \underline{21} & = & \underline{1176c.} \\
 & & 77 &) & \underline{2541c.} \\
 & & \text{Ans.} & & 33c.
\end{array}$$

(ART. **292**, p. 198.)

REMARK.—It is sometimes said that the mode of solving examples in Alligation by the "*linking*" process is "*arbitrary*" and "*blind*." It is believed, however, that it is perfectly philosophical and very simple.

By this process, however many ingredients enter into the mixture, *they always balance in pairs;* the gain derived from putting into the mixture a quantity of a low-priced article being offset by the loss sustained by putting in a quantity of an expensive article. The lines or "links" used in *connecting* the prices of the simples are merely for convenience of reference.

For illustration take Ex. 3, p. 198 of the Arithmetic, and we may have the following solutions:

BY ANALYSIS.

$$\text{7s.}\left\{\begin{array}{c|c|c|c} \text{s.} & \text{gal.} & \text{gal.} & \text{gal.} \\ 3 & \frac{1}{4} & & 6 \\ 5 & & \frac{1}{2} & 3 \\ 10 & & \frac{1}{3} & 2 \\ 13 & \frac{1}{6} & & 4 \end{array}\right\}\text{Ans.}$$

BY LINKING.

$$\text{7s.}\left\{\begin{array}{c|c} \text{s.} & \text{gal.} \\ 3 & 6 \\ 5 & 3 \\ 10 & 2 \\ 13 & 4 \end{array}\right\}\text{Ans.}$$

EXPLANATION.—By selling one gallon of the 3-shilling oil for 7 shillings we gain 4s., and ∴ to gain 1s. will require only $\frac{1}{4}$ of a gallon, which is placed at the right of the 3s. Again, by selling a gallon of the 13-shilling oil for 7s. we lose 6s., and ∴ to lose 1s. will require only $\frac{1}{6}$ of a gallon, which is placed at the right of the 13s. Hence $\frac{1}{4}$ of a gallon of the 3-shilling oil and $\frac{1}{6}$ of a gallon of the 13-shilling oil, or *any equimultiples* of these fractions, will make a mixture worth 7s. per gallon. In like manner it is evident that $\frac{1}{2}$gal. of the 5-shilling oil and $\frac{1}{3}$gal. of the 10-shilling oil, or any equimultiples of these fractions, will make a mixture of the required price per gallon. Now if $\frac{1}{4}$ and $\frac{1}{6}$ are multiplied by 24, the product of their denominators, we obtain 6 and 4, which are respectively placed in the Ans., at the right of the 3s. and 13s. in the above analysis; and if the $\frac{1}{2}$ and $\frac{1}{3}$ are multiplied by 6, the product of *their* denominators, we obtain 3 and 2, which are placed at the right of the 5s. and 10s. Thus a mixture worth 7s. per gallon is formed, by taking 6, 3, 2, and 4 gallons of oil at 3s., 5s., 10s., and 13s. per gallon, severally.

It will be seen that the 6 placed in the Ans. against the 3s. is the difference between 7, the mean rate, and 13; and the 4 placed against the 13 is the difference between 7 and 3. So in each couplet compared, the number obtained in the answer for *either term is the difference between the mean rate and the other term;* but by the "linking" process, Rule in Art. 290, we have precisely and necessarily the same result, and with much less labor.

ANOTHER SOLUTION.

3.

$$\overset{\text{s.}}{7}\left\{\begin{array}{rrcrcrl}
\overset{\text{s.}}{3} & \overset{\text{gal.}}{6} & \times & \overset{\text{s.}}{-4} & = & \overset{\text{s.}}{-24} & \\
5 & 3 & \times & -2 & = & \underline{-\;6} & \overset{\text{s.}}{-30}, \text{ deficiency.} \\
10 & 2 & \times & 3 & = & 6 & \\
13 & 2 & \times & 6 & = & \underline{12} & \\
 & & & & & & \underline{18}, \text{ surplus.}
\end{array}\right.$$

$$\begin{array}{rrl}
 & -12, & \text{deficiency.} \\
\text{Add oil at } \overset{\text{s.}}{13}\text{., } \overset{\text{gal.}}{2} \times \overset{\text{s.}}{6} = & \underline{12}, & \text{surplus.} \\
 & 0 &
\end{array}$$

Ans. 6gal. at 3s., 3gal. at 5s., 2gal. at 10s., and 4gal. at 13s.

EXPLANATION. — Having assumed 6, 3, 2, and 2 gallons at 3, 5, 10, and 13s., severally, we find that the mixture is not worth enough by 12s.; and it is readily seen that adding 2gal. at 13s. gives a mixture of the right quality.

NOTE. — The following examples in Alligation are solved only by the method explained in Art. 291, that being considered the most convenient mode, and the best for practical purposes.

4.

$$\begin{array}{ll}
\text{Assume} & 38\text{c.}\left\{\begin{array}{l}
\left.\begin{array}{rlrlr}
\overset{\text{lb.}}{2} & \text{at} & \overset{\text{c.}}{20}, & \text{gain} & \overset{\text{c.}}{36} \\
4 & \text{at} & 28, & \text{gain} & 40 \\
1 & \text{at} & 33, & \text{gain} & 5
\end{array}\right\} = 81\text{c. gain.} \\
\left.\begin{array}{rlrlr}
3 & \text{at} & 47, & \text{loss} & 27 \\
\underline{2} & \underline{\text{at}} & \underline{60,} & \underline{\text{loss}} & \underline{44} \\
1 & \text{at} & 28, & \text{loss} & 10
\end{array}\right\} = 81\text{c. loss.}
\end{array}\right. \\
\text{Abate} &
\end{array}$$

Ans. 2lb. at 20c., 3lb. at 28c., 1lb. at 33c., 3lb. at 47c., 2lb. at 60c.

5.

$$\begin{array}{ll}
\text{Assume} & \$1.60\left\{\begin{array}{l}
\left.\begin{array}{rlrlr}
\overset{\text{gal.}}{4} & \text{at} & \overset{\$}{0}, & \text{gain} & \overset{\$}{6.40} \\
2 & \text{at} & 1.50, & \text{gain} & .20
\end{array}\right\} = \$6.60 \text{ gain.} \\
\left.\begin{array}{rlrlr}
1 & \text{at} & 1.80, & \text{loss} & .20 \\
\underline{12} & \underline{\text{at}} & \underline{2.00,} & \underline{\text{loss}} & \underline{4.80} \\
4 & \text{at} & 2.00, & \text{loss} & 1.60
\end{array}\right\} = \$6.60 \text{ loss.}
\end{array}\right. \\
\text{Add} &
\end{array}$$

Ans. 4, 2, 1, and 16 gal. at 0, $1.50, $1.80, and $2.

		lb.		c.		c.	
6. Assume	12c.	8	at	4,	gain	64	= 82c. gain.
		3	at	7,	gain	15	
		3	at	11,	gain	3	
		4	at	15,	loss	12	= 82c. loss.
		5	at	20,	loss	40	
Add		2	at	15,	loss	6	
Add		3	at	20,	loss	24	

Ans. 8, 3, 3, 6, and 8 lb. at 4, 7, 11, 15, and 20c.

(Art. **293**, p. 199.)

		gal.		s.		s.	
2. Assume	10s.	40	at	0,	gain	400	= 425s. gain.
		5	at	6,	gain	20	
		5	at	9,	gain	5	
		8	at	15,	loss	40	= 425s. loss.
Add		77	at	15,	loss	385	

Ans. 5, 5, and 85 gal. at 6, 9, and 15s.

Note.—In solving examples in Art. 293, assume the *given quantity* of that kind whose quantity *is* given, and vary the others as convenience dictates.

		gal.		c.		c.	
3. Assume	30c.	63	at	50,	loss	1260	= 1290c. loss.
		10	at	33,	loss	30	
		8	at	25,	gain	40	= 1290c. gain.
		6	at	20,	gain	60	
Add		119	at	20,	gain	1190	

Ans. 10, 8, and 125 gal. at 33, 25, and 20c.

		lb.		c.		c.	
4. Assume	37c.	100	at	50,	loss	1300	= 1309c. loss.
		3	at	40,	loss	9	
		18	at	30,	gain	126	= 1309c. gain.
		4	at	24,	gain	52	
Add		87	at	24,	gain	1131	

Ans. 3, 18, and 91 lb. at 40, 30, and 24c.

5. Assume

$$\begin{array}{r}\text{carats.}\\21\end{array}\left\{\begin{array}{l}\begin{array}{rlrlr}\text{oz.}&&\text{carats.}&&\text{carats.}\end{array}\\ \left.\begin{array}{rlrlr}15&\text{at}&24,&\text{loss}&45\\3&\text{at}&22,&\text{loss}&3\end{array}\right\}=\overset{\text{carats.}}{48},\text{ loss.}\\ \left.\begin{array}{rlrlr}7&\text{at}&18,&\text{gain}&21\\6&\text{at}&16,&\text{gain}&30\end{array}\right\}=51,\text{ gain.}\end{array}\right.$$

$$\text{Abate}\qquad 1\ \text{at}\ 18,\ \text{loss}\ \ 3\ =\ 3,\ \text{loss.}$$

Ans. 3, 6, and 6oz. at 22, 18, and 16 carats.

(ART. 294. p. 201.)

2. Assume

$$37\tfrac{1}{2}\text{c.}\left\{\begin{array}{l}\begin{array}{rlrlr}\text{lb.}&&\text{c.}&&\text{c.}\end{array}\\ \left.\begin{array}{rlrlr}20&\text{at}&25,&\text{gain}&250\\12&\text{at}&31,&\text{gain}&78\end{array}\right\}=328\text{c. gain.}\\ \left.\begin{array}{rlrlr}2&\text{at}&40,&\text{loss}&5\\4&\text{at}&42,&\text{loss}&18\\2&\text{at}&45,&\text{loss}&15\\12&\text{at}&50,&\text{loss}&150\\4&\text{at}&70,&\text{loss}&130\\ \hline \text{Add}\quad 4&\text{at}&40,&\text{loss}&10\end{array}\right\}=328\text{c. loss.}\end{array}\right.$$

60, sum of proportional parts.

$$\left.\begin{array}{rcrcrcrl}\text{lb.}&&\text{lb.}&&\text{lb.}&&\text{lb.}&\\60&:&300&::&20&:&100&\text{at 25c.}\\60&:&300&::&12&:&60&\text{at 31c. and at 50c.}\\60&:&300&::&6&:&30&\text{at 40c.}\\60&:&300&::&4&:&20&\text{at 42c. and at 70c.}\\60&:&300&::&2&:&10&\text{at 45c.}\end{array}\right\}\text{Ans.}$$

The sum of the proportional parts, 60lb., is just $\frac{1}{5}$ of the required compound, 300lb.; ∴ each of the proportional parts is $\frac{1}{5}$ of the quantity which may be taken of that kind;

$$\text{Hence,}\quad\left.\begin{array}{rcrcrl}\text{lb.}&&&&\text{lb.}&\\20&\times&5&=&100&\text{at 25c.}\\12&\times&5&=&60&\text{at 31c.}\\6&\times&5&=&30&\text{at 40c.}\\4&\times&5&=&20&\text{at 42c.}\\2&\times&5&=&10&\text{at 45c.}\\12&\times&5&=&60&\text{at 50c.}\\4&\times&5&=&20&\text{at 70c.}\end{array}\right\}\text{Ans. by Analysis.}$$

3. Assume

$$\begin{array}{ll}
108\text{c.}\left\{\begin{array}{l}
\left.\begin{array}{rlrr}
\text{gal.} & & \text{c.} & \text{c.}\\
2 & \text{at} & 0,\ \text{gain} & 216\\
2 & \text{at} & 75,\ \text{gain} & 66\\
7 & \text{at} & 90,\ \text{gain} & 126\\
9 & \text{at} & 100,\ \text{gain} & 72
\end{array}\right\} = 480\text{c. gain.}\\
\begin{array}{rlrr}
6 & \text{at} & 124,\ \text{loss} & 96
\end{array}
\end{array}\right. & \\
\text{Add}\qquad\qquad\qquad \begin{array}{rlrr}
24 & \text{at} & 124,\ \text{loss} & 384
\end{array} & \Big\} = 480\text{c. loss.}
\end{array}$$

50, sum of proportional parts, equal to $\frac{1}{60}$ of the required compound; ∴ 60 times each of the proportional parts will form the mixture required; thus,

$$\left.\begin{array}{rcrcrlr}
\text{gal.} & & & & \text{gal.} & & \text{c.}\\
2 & \times & 60 & = & 120 & \text{at} & 0\\
2 & \times & 60 & = & 120 & \text{at} & 75\\
7 & \times & 60 & = & 420 & \text{at} & 90\\
9 & \times & 60 & = & 540 & \text{at} & 100\\
30 & \times & 60 & = & 1800 & \text{at} & 124
\end{array}\right\}\text{Ans.}$$

4. Assume

$$\begin{array}{ll}
16\text{s.}\left\{\begin{array}{l}
\left.\begin{array}{rlrr}
\text{sheep.} & & \text{s.} & \text{s.}\\
3 & \text{at} & 9,\ \text{gain} & 21\\
5 & \text{at} & 10,\ \text{gain} & 30\\
9 & \text{at} & 15,\ \text{gain} & 9
\end{array}\right\} = 60\text{s. gain.}\\
\begin{array}{rlrr}
10 & \text{at} & 18,\ \text{loss} & 20\\
2 & \text{at} & 24,\ \text{loss} & 16
\end{array}
\end{array}\right. & \\
\text{Add}\qquad\qquad\qquad \begin{array}{rlrr}
3 & \text{at} & 24,\ \text{loss} & 24
\end{array} & \Big\} = 60\text{s. loss.}
\end{array}$$

32 = $\frac{1}{5}$ of the required flock, ∴

$$\left.\begin{array}{rcrcrlr}
\text{sheep} & & & & \text{sheep.} & & \text{s.}\\
3 & \times & 5 & = & 15 & \text{at} & 9\\
5 & \times & 5 & = & 25 & \text{at} & 10\\
9 & \times & 5 & = & 45 & \text{at} & 15\\
10 & \times & 5 & = & 50 & \text{at} & 18\\
5 & \times & 5 & = & 25 & \text{at} & 24
\end{array}\right\}\text{Ans.}$$

5. Assume

$$\begin{array}{ll}
\begin{array}{c}\text{carats.}\\ 21\end{array}\left\{\begin{array}{l}
\left.\begin{array}{rlrr}
\text{oz.} & & \text{carats.} & \text{carats.}\\
6 & \text{at} & 18,\ \text{gain} & 18\\
1 & \text{at} & 20,\ \text{gain} & 1
\end{array}\right\} = \begin{array}{c}\text{carats.}\\ 19\ \text{gain.}\end{array}\\
\begin{array}{rlrr}
2 & \text{at} & 23,\ \text{loss} & 4\\
3 & \text{at} & 24,\ \text{loss} & 9
\end{array}
\end{array}\right. & \\
\text{Add}\qquad\qquad\qquad \begin{array}{rlrr}
3 & \text{at} & 23,\ \text{loss} & 6
\end{array} & \Big\} = 19\ \text{loss.}
\end{array}$$

15 = $\frac{1}{5}$ of the required compound, ∴

$$\left.\begin{array}{ccccccc} \text{oz.} & & & & \text{oz.} & & \text{carats.} \\ 6 & \times & 2 & = & 12 & \text{at} & 18 \\ 1 & \times & 2 & = & 2 & \text{at} & 20 \\ 5 & \times & 2 & = & 10 & \text{at} & 23 \\ 3 & \times & 2 & = & 6 & \text{at} & 24 \end{array}\right\} \text{Ans.}$$

SINGLE POSITION.

3. Assume 6 scholars.
As many more, 6 — 300
½ as many more, 3 — Subtract 36½
⅓ as many more, 2 — 263½
17 : 263½ :: 6 : 93, Ans.

ANALYSIS. — He had 1 school; if he had as many more, and ½ as many more, and ⅓ as many more he would have 2$\frac{5}{6}$ schools. Then if he had 36½ more scholars he would have 300; but, not having the 36½ scholars, he would have 300 — 36½ = 263½ scholars; ∴ 263½ scholars = 2$\frac{5}{6}$ schools, and ∴ 263½ ÷ 2$\frac{5}{6}$ = 93, No. of scholars in school, Ans.

4. Let \$36 = annual income of each.
12 = ⅓ of \$36, A's savings.
24 = A's expenses.
48 = twice \$24, B's expenses.
\$48 — \$36 = \$12, B's debt in 1 year.
\$12 : \$120 :: \$36 : \$360, Ans.

NOTE. — If B's debt is \$480 in 4 years, it is \$120 in 1 year. A similar abbreviation is made in several examples in the Positions.

ANALYSIS. — If A saves ⅓ of his income, he spends ⅔, and ∴ B spends $\frac{4}{3}$, and is consequently in debt to the amount of ⅓ of his income. Hence \$120 is ⅓ and \$360 is the whole of his annual income.

5. Assume 20 years. ⅖ of 20 = 8.
8 × 9 = 72
⅗ of 20 = 12
60 : 150 :: 20yr. : 50yr., Ans.

6. Assume 40.
$\frac{7}{8}$ of 40 = 35
$\frac{1}{5}$ of 40 = 8
Excess = 27 : 81 :: 40 : 120, Ans.

7. Assume $1 for principal.
Amount of $1 for 3 years = $1.191016 (See Table, p. 145 of Arithmetic).
∴ $1.191016 : $11910.16 :: $1 : $10000, principal.
$11910.16 — $10000 = $1910.16, Ans.

8. Assume $12 for harness; then
$\frac{5}{4}$ of $12 = $15 for horse; and
$\frac{4}{3}$ of $15 = $20 for chaise.
$47 : $470 :: $12 : $120, price of harness.
$47 : $470 :: $15 : $150, price of horse.
$47 : $470 :: $20 : $200, price of chaise.

DOUBLE POSITION.

2.	First assume		40£	Next assume		48£
	$\frac{1}{8}$ of 40£	=	5£	$\frac{1}{8}$ of 48£	=	6£
	A's expenses	=	35£	A's expenses	=	42£
			30£			30£
	B's expenses	=	65£	B's expenses	=	72£
	B's income	=	40£	B's income	=	48£
	B's annual debt	=	25£	B's annual debt	=	24£

25£ — 5£ = 20£, 1st error. 24 £ — 5£ = 19£, 2d error.
2d assumpt., 48 1st assumpt., 40
960£ 20, 1st error. 760£
760£ 19, 2d error.
Dif. of prod., 200£ ÷ 1 = 200£, Ans.

Note. — B's debt, being 40£ in 8 years, is 5£ in 1 year.

Remark. — These examples may be solved in various ways. The method adopted in the Key will be understood when examined *in connection with the example.*

3. First assume, Next assume,

$$\begin{array}{llrr|llrr}
 & \text{gal.} & & \text{gal.} & & \text{gal.} & & \text{gal.} \\
\text{Less} & 6 & \text{Greater} & 18 & \text{Less} & 10 & \text{Greater} & 30 \\
 & 4 & & \underline{4} & & 4 & & \underline{4} \\
 & — & & 14 & & — & & 26 \\
 & 2 \times 4 = & & \underline{8} & & 6 \times 4 = & & \underline{24} \\
\text{1st error} & & = & 6 & \text{2d error} & & = & 2 \\
\text{2d assumption,} & & & \underline{10} & \text{1st assumption,} & & & \underline{6} \\
 & & & 60 & & & & 12 \\
 & & & \underline{12} & & & &
\end{array}$$

6, 1st error.
$\underline{2}$, 2d error.

Dif. of products, 48 ÷ 4 = 12gal., Less
12gal. × 3 = 36gal., Greater } Ans.

4\.

First assume	$330	Next assume	$450
A saves ⅓,	110	A saves ⅓,	150
A spends ⅔,	220	A spends ⅔,	300
	200		200
B spends	420	B spends	500
B's income,	330	B's income,	450
B's annual debt,	90	B's annual debt,	50

After A pays B's debt,
A has $150 — $50 = $100 in the 2d case.
$100 — $50 = $50, 2d error.
1st assumption, $330
$16500

And A has $110 — $90 = $20 in the 1st case.
$50 — $20 = $30, 1st error.
2d assumption, $450
$13500 30, 1st error.
16500 50, 2d error.
Sum of products, $30000 ÷ 80 = $375, Ans.

NOTE.—The first assumed number was too small, for A, after paying B's debt, had not so much money left as the example requires; the second assumed number is too large, for A has more money left than is required; ∴ the *sum* of the products is divided by the *sum* of the errors, as the rule requires.

5. First assume, Next assume,
7)84 7)98
12 14
Subtract 10 10
2 4
Multiply by 3 3
24 — 6 = 18, 1st error. 24 — 12 = 12, 2d error.
2d assumption, 98 1st assumption, 84
1764 18, 1st error. 1008
1008 12, 2d error.
Dif. of products, 756 ÷ 6 = 126, Ans.

6. First assume, Next assume,
lb. lb.
Body = 20 30
Head = 14 14
∴ Tail = 20, i.e. head + $\frac{3}{10}$ body. 23
H'd & tail = 34 37
Body = 20 30
14, 1st error. 7, 2d error.
2d assumpt., 30 1st assumpt., 20
420 14, 1st error. 140
140 7, 2d error.
Dif. of prod., 280 ÷ 7 = 40lb., body.
2
80lb., Ans.

7. First suppose he was absent 20 days.
Then he worked 50 — 20 = 30 days.
\$1.50 × 30 = \$45, sum earned.
\$1.75 × 20 = \$35, sum forfeited.
\$10, sum received.
\$42.50 — \$10 = \$32.50, 1st error.

Next suppose he was absent 15 days.
Then he worked 50 — 15 = 35 days.

$1.50 × 35 = $52.50, sum earned.
$1.75 × 15 = $26.25, sum forfeited.
$26.25, sum received.

$42.50 — $26.25 = $16.25, 2d error.

$32.50 × 15 = $487.50, 1st error × 2d assumed No.
$16.25 × 20 = $325.00, 2d error × 1st assumed No.
$16.25) $162.50 (10, Ans.
$162.50

8. First assume 3 horses at $50 = $150
∴ 9 oxen at $80 = $720
and 18 cows at $25 = $450
Total cost, $1320

$2640 — $1320 = $1320, 1st error.

Next assume 4 horses at $50 = $200
∴ 12 oxen at $80 = $960
and 24 cows at $25 = $600
Total cost, $1760

$2640 — $1760 = $880, 2d error.

$1320 × 4 = $5280
$880 × 3 = $2640
$440) $2640
No. of horses, 6

6 horses
18 oxen } Ans.
36 cows

9.

First assume,			Next assume,
First horse worth	$30		$40
∴ saddle	15		20
If $45 is $\frac{3}{10}$,	3)45		3)60
Then $\frac{1}{10}$ is	15		20
	10		10
∴ 2d horse is worth	150		200
Saddle	15		20
	165		220
$50 + 3 times 1st horse	140		170
1st error	25	2d error	50

$\$50 \times 30 = \$1500$		\$20, 1st horse.
$25 \times 40 = 1000$		10, saddle.
25	) 500	3)30
1st horse,	20	$\$10 \times 10 = \$100$, Ans.

INVOLUTION AND POWERS.

2. $6^6 = 6 \times 6 \times 6 \times 6 \times 6 \times 6 = 46656$, Ans.
3. $6^4 \times 6^2 = 6^{4+2} = 6^6 = 46656$, Ans.
4. $3^5 \times 3^3 = 3^{5+3} = 3^8 = 6561$, Ans.
5. $(\frac{6}{21})^3 = (\frac{2}{7})^3 = \frac{2}{7} \times \frac{2}{7} \times \frac{2}{7} = \frac{8}{343}$, Ans.
6. $(3\frac{1}{4})^2 = (\frac{13}{4})^2 = \frac{13}{4} \times \frac{13}{4} = \frac{169}{16} = 10\frac{9}{16}$, Ans.
7. $(\frac{7}{3})^3 = \frac{343}{27}$.
8. $.25^2 = .25 \times .25 = .0625$.
9. $.006^3 = .000000216$.
10. $1^6 = 1$.
11. $7^6 \div 7^4 = 7^{6-4} = 7^2 = 49$.
12. $9^8 \div 9^5 = 9^{8-5} = 9^3 = 729$.
13. $10^{19} \div 10^{18} = 10^{19-18} = 10$.
14. $2 \times 3 = 6$.
15. $2 \times 3 - 1 = 5$.
16. $2 \times 3 - 2 = 4$.
17. $2 \times 5 = 10$.
18. $2 \times 5 - 4 = 6$.

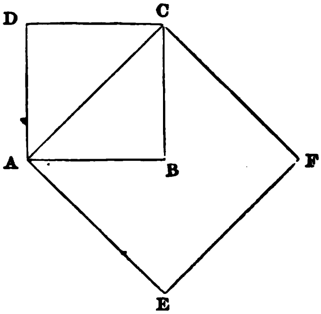
SQUARE ROOT.

15.	2401	23.	13.302+	27.	13.70933+
16.	6561	24.	5.04648+	28.	7.06229+
17.	7776	25.	19.90439+	37.	.91657+
18.	16807	26.	29.54555+	38.	.61395+

APPLICATION OF SQUARE ROOT.

Remark.—The solution of examples in the *Application of the Square Root* is often made more simple by the construction of geometrical figures, showing the relations of the given conditions to the things required. The more skillfully the figure is constructed the more readily will the example be solved.

1. In 10 acres there are $160 \times 10 = 1600$sq. rd. $\sqrt{1600} = 40$, Ans.

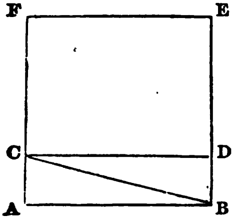


2. ABCD is a square; $\therefore$ ABC is a right-angled triangle, in which AB and BC are equal. Hence (Art. 330, Prop. 6), $AC^2 = AB^2 + BC^2 = 2\,AB^2$; but AB^2, i.e. the square ABCD contains 20 acres; $\therefore$ AC^2, i.e. the square AEFC, contains 40 acres $= 43560$sq. ft. $\times 40 = 1742400$sq.ft. Hence AC $= \sqrt{1742400} = 1320$ft., Ans.

3. The stump, 21ft., is the perpendicular, the line on the ground, 28ft., is the base, and the part of the tree broken off is the hypothenuse of a right-angled triangle; hence $\sqrt{21^2 + 28^2} = \sqrt{1225} = 35$ft., Ans.

4. The hight of the fort, 24ft., the length of the ladder, 40ft., and the width of the stream are the perpendicular, the hypothenuse, and the base of a right-angled triangle; $\therefore$ $\sqrt{40^2 - 24^2} = \sqrt{1024} = 32$ft., Ans.

5. $\sqrt{100^2 - 80^2} = \sqrt{3600} = 60$ft., Ans.



6. The triangle ABC contains $1\frac{1}{8}$ acres, $\therefore$ the rectangle ABDC contains 2 times $1\frac{1}{8} = 2\frac{1}{4}$ acres, and, from the nature of the question, if the square ABEF is made, *it* will contain $4\frac{4}{9}$ times $2\frac{1}{4}$ acres $= 10$ acres $= 1600$sq. rd.; $\therefore$ AB $= \sqrt{1600} = 40$rd., and AC $= 40 \div 4\frac{4}{9} = 9$rd., and CB $= \sqrt{40^2 + 9^2} = \sqrt{1681} = 41$rd. Hence the distance round ABC is $40 + 9 + 41 = 90$rd.; $\therefore$ $66\frac{2}{3}$c. $\times 90 = \$60$, Ans.

Another Solution.—If AC, in the above figure, is 1 rod, then AB $= 4\frac{4}{9}$rd., and the triangle ABC contains $\frac{1}{2}$ of $4\frac{4}{9}$sq. rd. $= 2\frac{2}{9}$sq. rd.; but the given triangular field, which is *similar* to ABC, contains $1\frac{1}{8}$a. $=$ 180sq. rd.; $\therefore$ (Art. 330, Prop. 9) $2\frac{2}{9} : 180 :: AC^2 = 1 : 81$, and $\therefore \sqrt{81} = 9$, is the length in rods of the shortest or perpendicular side of the given triangular field, and $\therefore$ the base is $4\frac{4}{9}$ times 9rd. $=$ 40rd. The hypothenuse and cost of fencing the field may be found as in the first solution.

7. One ship sails 24 times 8 miles $=$ 192m. and the other 24 times 10m. $=$ 240m.;

$$\therefore \sqrt{192^2 + 240^2} = \sqrt{94464} = 307.34996+\text{m., Ans.}$$

NOTE. — This example may be solved by finding how far apart the ships are in one hour, and then multiplying this distance by 24.

8. $50^2 = 2500$; (Art. 330, Prop. 4), $.785398 \times 2500 = 1963.495$; $\sqrt{1963.495} = 44.31134+$ft., Ans.

9. The areas of the two ponds are to each other in the ratio 1 to 16; $\therefore$ (Art. 330, Prop. 5),

$1 : 16 :: 10^2$: *square* of required diameter,

and, evolving, $1 : 4 :: 10$: required diameter $=$ 40rd., Ans.

10. $62.83184 \div 3.141592 = 20$, diameter (Art. 330, Prop. 3); $\sqrt{\frac{1}{4}} = \frac{1}{2}$; $\frac{1}{2}$ of 20ft. $=$ 10ft., Ans.

11. $\sqrt{546121} = 739$, Ans.

12. Hypothenuse $=$ 35ft., perpendicular $=$ 15ft., $\therefore$ base $= \sqrt{35^2 - 15^2} = \sqrt{1000} = 31.622776+$ft. $= \frac{1}{2}$ width of the house, and 31.622776ft. $\times 2 =$ 63.24555+ft., Ans.

13. $25^2 + 20^2 = 1025 =$ the square of the diagonal of the *floor;* then the square of this diagonal $+$ the square of the hight of the room, 12ft., $=$ the *square* of the required line; i.e. *the sum of the squares of the three dimensions of any rectangular body is the square of the diagonal of that body;* $\therefore$

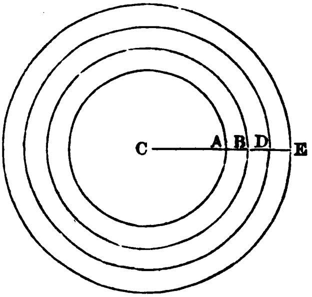
$$\sqrt{25^2 + 20^2 + 12^2} = \sqrt{1169} = 34.19064\text{ft., Ans.}$$

14. $(\frac{9}{4})^2 : (\frac{3}{4})^2$ or $3^2 : 1^2 ::$ 5h. $=$ 300m. : $33\frac{1}{3}$m., Ans.

15. $(\frac{1}{5})^2 : (\frac{1}{2})^2$, i.e. $\frac{1}{25} : \frac{1}{4} ::$ 1800lb. : 11250lb., Ans.

16. From the foot of the statue to the inner edge of the canal is 22ft., and from the foot of the statue to the outer edge is $\sqrt{61^2 - 11^2} = \sqrt{3600} =$ 60ft.; $\therefore$ the width of the canal is 60ft. $-$ 22ft. $=$ 38ft., Ans.

17. The whole stone is to be divided into 4 equal parts, one part represented by the circle whose radius is CA, another part by the circular ring whose width is AB, etc., etc.



The radius CE $=$ 18 inches, and if the whole stone is represented by 4 the smallest circle will be represented by 1, the circle whose radius is CB will be represented by 2, and the circle whose radius is CD, by 3; $\therefore$ (Art. 330, Prop. 5),

$4 : 1 :: CE^2 = 18^2 : CA^2 = 81$; $\therefore CA = \sqrt{81} = 9$in., 4th Ans.

$4 : 2 :: CE^2 = 18^2 : CB^2 = 162$; $\therefore CB = \sqrt{162} = 12.72792$; $CB - CA = AB$, i.e. 12.72792in. — 9in. $=$ 3.72792in., 3d Ans.

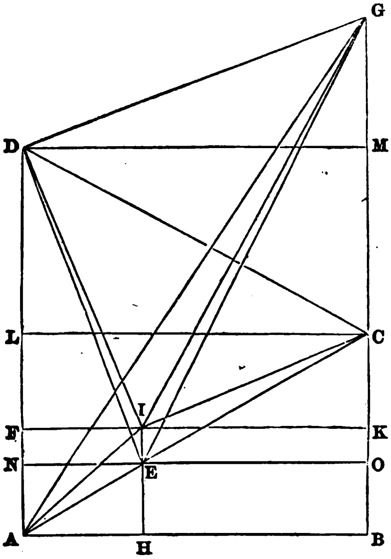
$4 : 3 :: CE^2 = 18^2 : CD^2 = 243$; $\therefore CD = \sqrt{243} = 15.58845$; $CD - CB = BD$, i.e. 15.58845in. — 12.72792in. $=$ 2.86053in., 2d Ans.

$CE - CD = DE$, i.e. 18in. — 15.58845in. $=$ 2.41155in., 1st Ans.

Remark.—In solving Ex. 17 it is necessary to find the length of the radius CB before finding the length of AB, for the proportion $4 : 2 :: CE^2 : CB^2$, is based on the principle (Art. 330, Prop. 5) that the areas of *similar* figures are to each other as the squares of their homologous sides; but *circles and circular rings are not similar figures.* For a like reason it is necessary to find CD in order to find BD. Analogous remarks may be made with reference to the solutions of examples 6 and 11 in the *Application of the Cube Root.*

The solutions of examples 18, 19, and 20 are based almost exclusively on the properties of right-angled triangles, and are perfectly simple, though somewhat long.

18. Since AC = 300ft. and the angle CAB = 30° (Art. 330, Prop. 10), BC = 150ft. = AL = LD. Again, since AE = 100ft., HE = 50ft. = BO = AN; HI = 50ft. + 10ft. = 60ft. = BK = AF; KC = 150ft. — 60ft. = 90ft.; BG = 150ft. + 250ft. = 400ft.; KG = 400ft. — 60ft. = 340ft.; MG = 400ft. — 300ft. = 100ft.; ND = 300ft. — 50ft. = 250ft.; FD = 300ft. — 60ft. = 240ft.



$AB = \sqrt{AC^2 - BC^2} = \sqrt{300^2 - 150^2} = \sqrt{67500} = 259.80762$, Ans. to (6).

$NE = FI = \frac{1}{3} AB = 86.60254$ft.; $EO = IK = \frac{2}{3} AB = 173.20508$ft.

$$\left.\begin{array}{l} IG = \sqrt{KG^2 + IK^2} = \sqrt{340^2 + 173.20508^2} = \\ \quad \sqrt{145599.9997378064} = 381.57568\text{ft.} \\ ID = \sqrt{FD^2 + IF^2} = \sqrt{240^2 + 86.60254^2} = \\ \quad \sqrt{65099.9999344516} = 255.14701\text{ft.} \end{array}\right\} \text{Ans. to (1).}$$

$$\left.\begin{array}{l} IC = \sqrt{KC^2 + IK^2} = \sqrt{90^2 + 173.20508^2} = \\ \quad \sqrt{38099.9997378064} = 195.19221\text{ft.} \\ IA = \sqrt{IH^2 + AH^2} = \sqrt{60^2 + 86.60254^2} = \\ \quad \sqrt{11099.9999344516} = 105.35653\text{ft.} \end{array}\right\} \text{Ans. to (2).}$$

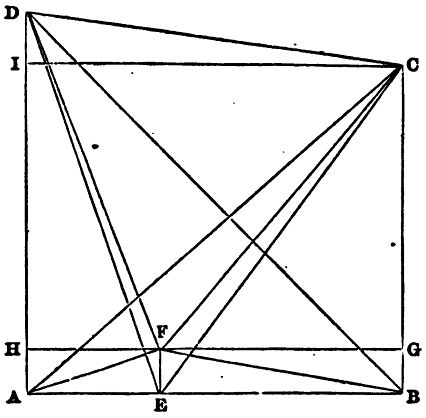
$$\left.\begin{array}{l} DG = \sqrt{GM^2 + DM^2} = \sqrt{100^2 + 259.80762^2} \\ \quad = \sqrt{77499.9994100644} = 278.38821\text{ft.} \end{array}\right\} \text{Ans. to (3).}$$

Since CL, the perpendicular from C to AD, *bisects* AD, it follows (converse of Art. 330, Prop. 1) that DC = AC = 300ft.; or DC may be found as follows:

$$\left.\begin{array}{l} DC = \sqrt{LD^2 + LC^2} = \sqrt{150^2 + 67500} = \\ \quad \sqrt{90000} = 300\text{ft.} \\ GA = \sqrt{AB^2 + BG^2} = \sqrt{67500 + 400^2} = \\ \quad \sqrt{227500} = 476.9696\text{ft.} \end{array}\right\} \text{Ans. to (4).}$$

$$\left.\begin{array}{l} GE = \sqrt{GO^2 + OE^2} = \sqrt{350^2 + 173.20508^2} = \\ \quad \sqrt{152499.9997378064} = 390.51248\text{ft.} \\ DE = \sqrt{DN^2 + NE^2} = \sqrt{250^2 + 86.60254^2} = \\ \quad \sqrt{69999.9999344516} = 264.57513\text{ft.} \end{array}\right\} \text{Ans. to (5).}$$

19. $$\left.\begin{array}{l} FC = \sqrt{FG^2 + GC^2} = \sqrt{200^2 + 240^2} = \\ \quad \sqrt{97600} = 312.40998\text{ft.} \\ FD = \sqrt{FH^2 + HD^2} = \sqrt{100^2 + 290^2} = \\ \quad \sqrt{94100} = 306.75723\text{ft.} \end{array}\right\} \text{Ans. to (1).}$$



$$\left.\begin{array}{l} FB = \sqrt{EB^2 + EF^2} = \sqrt{200^2 + 10^2} = \\ \quad \sqrt{40100} = 200.24984\text{ft.} \\ FA = \sqrt{EA^2 + EF^2} = \sqrt{100^2 + 10^2} = \\ \quad \sqrt{10100} = 100.49875\text{ft.} \end{array}\right\} \text{Ans. to (2).}$$

$$\left.\begin{array}{l} CD = \sqrt{CI^2 + ID^2} = \sqrt{300^2 + 50^2} = \\ \quad \sqrt{92500} = 304.13812\text{ft.} \end{array}\right\} \text{Ans. to (3).}$$

$$\left.\begin{array}{l} DB = \sqrt{AB^2 + AD^2} = \sqrt{300^2 + 300^2} = \\ \quad \sqrt{180000} = 424.26406\text{ft.} \\ CA = \sqrt{AB^2 + BC^2} = \sqrt{300^2 + 250^2} = \\ \quad \sqrt{152500} = 390.51248\text{ft.} \end{array}\right\} \text{Ans. to (4).}$$

$$\left.\begin{array}{l} CE = \sqrt{EB^2 + BC^2} = \sqrt{200^2 + 250^2} = \\ \quad \sqrt{102500} = 320.15621\text{ft.} \\ DE = \sqrt{AE^2 + AD^2} = \sqrt{100^2 + 300^2} = \\ \quad \sqrt{100000} = 316.22776\text{ft.} \end{array}\right\} \text{Ans. to (5).}$$

$AB = 300$ft., Ans. to (6).

REMARK.—Example 20 is sometimes solved by setting a plant in the center of the plat, and then setting 6 plants in the angles of a hexagon around the central plant, and then arranging 12 plants in the perimeter of a second hexagon around the first, and so on; but this solution is longer than the following, and requires much more knowledge of Geometry.

20. The plants on the lines *ab*, *de*, *fg*, etc. (see figure in the Arithmetic), are so placed that the plants at *c*, *v*, and *z*, those at *v*, *z*, and *y*, etc., shall be at the vertices of equilateral triangles, each of whose sides is 10 inches long. In this way the *rows* of plants are *less* than 10 inches apart, while the *plants themselves are not within* 10 *inches of each other*, and thus more plants are set in the circle than could be *if the rows were* 10 *inches apart.*

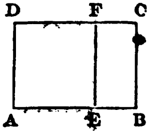
Since the rows are equally distant from each other, and since $cx^2 = 75$, $\therefore$ (Art. 330, Prop. 7) $cy^2 = 4$ times $75 = 300$, and the square of the *next* perpendicular will be 9 times $75 = 675$, etc., etc., while the hypothenuses *cd*, *cf*, etc. are severally 80 inches in length, they being radii of the same circle. Having found the lengths of the several lines *dx*, *fy*, etc. it will be necessary to subtract 5 inches from the 1st, 3d, 5th, etc. lines, because the central plants on those lines do not stand on the line *ch*, but 5 inches to the left or to the right of *ch*.

Now, adopting the suggestions in the Arithmetic, and omitting fractions and remainders, which cannot be used, we have $dx = \sqrt{cd^2 - cx^2} = \sqrt{80^2 - 75} = \sqrt{6400 - 75} = \sqrt{6325} = 79$; $79 - 5 = 74$; $74 \div 10 = 7$; i.e. there are 7 *spaces* between the plants on the line *dv*, and ∴ there may be 8 plants on the line *dv* and also 8 on *ze*, and consequently the number of plants on *de* will be 16

In like manner $fy = \sqrt{80^2 - 300} = \sqrt{6400 - 300}$ $= \sqrt{6100} = 78$; $78 \div 10 = 7$; i.e. 7 plants may be set on the line *fg at the left* of *y*, 7 *at the right* of *y*, and 1 *at y*, and consequently the number on *fg* is 15

3d line above $ac = \sqrt{6400 - 675} = \sqrt{5725} = 75$;
$75 - 5 = 70$; $70 \div 10 = 7$; $(7 + 1) \times 2 =$ 16
4th line $= \sqrt{6400 - 1200} = \sqrt{5200} = 72$;
$72 \div 10 = 7$; $7 \times 2 + 1 =$ 15
5th line $= \sqrt{6400 - 1875} = \sqrt{4525} = 67$;
$67 - 5 = 62$; $62 \div 10 = 6$; $(6 + 1) \times 2 =$ 14
6th line $= \sqrt{6400 - 2700} = \sqrt{3700} = 60$;
$60 \div 10 = 6$; $6 \times 2 + 1 =$ 13
7th line $= \sqrt{6400 - 3675} = \sqrt{2725} = 52$;
$52 - 5 = 47$; $47 \div 10 = 4$; $(4 + 1) \times 2 =$ 10
8th line $= \sqrt{6400 - 4800} = \sqrt{1600} = 40$;
$40 \div 10 = 4$; $4 \times 2 + 1 =$ 9
9th line $= \sqrt{6400 - 6075} = \sqrt{325} = 18$;
$18 - 5 = 13$; $13 \div 10 = 1$; $(1 + 1) \times 2 =$ 4
Adding, we find the No. of plants *above ab* is 112
∴ evidently the No. *below ab* is also 112
To these add the No. *on ab*, viz. 17
and we have for the Ans. 241

NOTE. — Should we attempt to find a *tenth* line, in the above solution, carrying out the same method, we should have 10th line $= \sqrt{6400 - 7500} = \sqrt{-1100}$, a number whose value cannot be found, since *no negative number* can have a *square root.*



21. If the rectangle ABCD has its length to its breadth as 3 to 2, and contains 60 acres, and if $\frac{1}{3}$ of it is cut off by the line EF, there will remain a square, AEFD, containing 40 acres = 6400sq. rd.;

∴ AD $= \sqrt{6400} = 80$rd., width
and 2 : 3 :: AD = 80rd. : AB = 120rd., length

22. $380192 \times 2 = 760384$; $\sqrt{760384} = 872$, Ans. (Art. 257, a).

23. $\sqrt{375^2 - 225^2} = \sqrt{140625 - 50625} = \sqrt{90000} =$ 300ft., Ans.

24. $\sqrt{25^2 - 24^2} = \sqrt{49} =$ 7ft, distance to one side.
$\sqrt{25^2 - 16^2} = \sqrt{369} =$ 19.20937ft., dist. to other side.
$\therefore$ width of street $=$ 26.20937ft., Ans.

25. $480 \div 10 = 48$, No. of square inches in one end of the box. Now, if $\frac{1}{4}$ of the end of the box is cut off by a line parallel to the top of the box, there will remain a square containing 36 square inches;

$\therefore$ $\sqrt{36} =$ 6 inches, width of box;
and 3 : 4 :: 6in. : 8in., hight of box;
and $\sqrt{10^2 + 8^2 + 6^2} = \sqrt{200} =$ 14.14213in., Ans.

26. 40£ 16s. 9d. = 9801d.; $\sqrt{9801} = 99$, Ans.

27. A *square* field of the *same width* as the given field would contain $\frac{2}{5}$ of 25 acres = 10 acres = 1600sq. rd.;

$\therefore$ $\sqrt{1600} =$ 40rd., width of field;
and 2 : 5 :: 40rd. : 100rd., length of field;
140rd. $\times 2 =$ 280rd., length of fence;
50c. $\times 280 =$ \$140, Ans.

28. If the orchard is cut into 3 equal parts, each part will be a square containing 400 trees; $\therefore$ $\sqrt{400} = 20$, No. of rows, and 3 times 20 = 60, No. of trees in a row. Hence there are 19 *spaces* between the *rows* and 59 spaces between the *trees in a row*.

$19 \times 30 + 20 =$ 590ft., width of field;
$59 \times 30 + 20 =$ 1790ft., length of field;
$1790 \times 590 =$ 1056100sq. ft. in field;
$1056100 \div 9 =$ 117344yd. 4ft.;
$117344 \div 30\frac{1}{4} =$ 3879rd. $4\frac{1}{4}$yd.; $\frac{1}{4}$yd. = 2ft. 36in.;
$3879 \div 40 =$ 96r. 39rd.; $96 \div 4 =$ 24a.

Now, if the 2ft. obtained from the $\frac{1}{4}$yd. are put with the 4ft. we have 6ft.; $\therefore$ 24a. 39rd. 4yd. 6ft. 36in., Ans.

29. $\sqrt{121 \times 49} = 11 \times 7 = 77$rd., Ans.

30. 10a. $=$ 1600sq. rd.; $1600 \div .785398 = 2037.183695$;
$\sqrt{2037.183695} = 45.135$, diameter (Art. 330, Prop. 4);
$45.135 \times 3.141592 = 141.7957$rd., circumference (Art. 330, Prop. 3);
$\sqrt{1600} = 40$rd., 1 side of a square 10-acre lot;
$40 \times 4 = 160$rd., 4 sides of a square 10-acre lot;
$160 - 141.7957 = 18.204$rd., difference of fence;
50c. $\times\ 18.204 = \$9.102$, Ans.

31. The perpendicular $=$ 30in. (Art. 330, Prop. 10);
$\sqrt{60^2 - 30^2} = \sqrt{2700} \doteq 51.96152$, base;
$60 - 51.96152 = 8.03848$in., Ans.

32. $6 \times 5^2 \times 10 = 1500$, Ans. (Art. 330, Prop. 5).

33. $\sqrt{12^2 + 12^2 + 12^2}$ or $\sqrt{12^2 \times 3} = \sqrt{432} = 20.7846$ft., Ans.

CUBE ROOT.

9.	147	18.	1.91293+	30.	.94991+
10.	2002	19.	2.41014+	32.	$\frac{5}{3} = 1\frac{2}{3}$
11.	333	20.	2.6684+	33.	1.77183+
12.	222	21.	3.03658+	34.	178.50555+
13.	726	23.	3.84821+	35.	7.4
16.	1.44224+	25.	41.87469+	36.	1.67359+
17.	1.5874+	26.	4.2	37.	$\frac{4}{42}$ or $\frac{2}{21}$

APPLICATION OF THE CUBE ROOT.

1. $1 \div \frac{1}{4} = 4$; $4^3 = 64$, Ans.
or, $(\frac{1}{4})^3 : 1^3 :: 1 : 64$, Ans. (Art. 336, Prop. 3).

2. $25 \div 5 = 5$; $5^3 = 125$, Ans.
or, $5^3 : 25^3 :: 1 : 125$. Ans.

3. $886144 \div 7912 = 112$; $112^3 = 1404928$, Ans.
or, $7912^3 : 886144^3 :: 1 : 1404928$, Ans.

4. $20 \div 5 = 4$; $4^3 \times 16 = 1024$lb., Ans.
or, $5^3 : 20^3 :: 16$lb. : 1024lb., Ans.

5. $2700 \div 100 = 27$; $\sqrt[3]{27} = 3$in., Ans.
or, $100 : 2700 :: 1^3 : 27$; $\sqrt[3]{27} = 3$in., Ans.

6. If a cone is cut by a plane parallel to the base, the part next the vertex is itself a cone, and is similar to the whole cone. In solving this example, first cut off $\frac{1}{4}$ of the cone, and find its hight for the 1st Ans. Then cut off $\frac{1}{2}$ of the whole cone, and find *its* hight; subtract the hight of the $\frac{1}{4}$ from the hight of the $\frac{1}{2}$, and the remainder will be the 2d Ans. Proceed in a similar manner for the 3d Ans. If the whole cone is represented by 4, A's part will be represented by 1; A's and B's, by 2; A's, B's, and C's, by 3; ∴

$4 : 1 :: 16^3 : 1024$, the *cube* of the hight of A's part;
$\sqrt[3]{1024} = 10.07936$, 1st Ans.
$4 : 2$ or $2 : 1 :: 16^3 : 2048$; $\sqrt[3]{2048} = 12.6992$;
$12.6992 - 10.07936 = 2.61984$, 2d Ans.
$4 : 3 :: 16^3 : 3072$; $\sqrt[3]{3072} = 14.53696$;
$14.53696 - 12.6992 = 1.83776$, 3d Ans.
$16 - 14.53696 = 1.46304$, 4th Ans.

7. A bushel equals 8 half pecks;

$$\left.\begin{array}{l}\sqrt[3]{8} = 2;\ 9\frac{1}{4} \times 2 = 18\frac{1}{2}\text{in., diameter}\\ 4 \times 2 = 8\text{in., depth}\end{array}\right\}\text{Ans.}$$

8. If the hight were no greater than the width the bin would contain only $\frac{1}{2}$ of $327680 = 163840$c. in.; then if the length also were only the same as the width it would contain only $\frac{1}{5}$ of $163840 = 32768$c. in.; ∴

$$\left.\begin{array}{l}\sqrt[3]{32768} = 32\text{in., width}\\ 32 \times 2 = 64\text{in., hight}\\ 32 \times 5 = 160\text{in., length}\end{array}\right\}\text{Ans.}$$

9. $144 \times 36 \times 9 = 46656$; $\sqrt[3]{46656} = 36$in., Ans.

REMARK.—The above solution of Ex. 9 is the most obvious one, but the work may be more easily performed as follows:

Resolving 36 into the two factors 12 and 3, combine the 12 with 144 and the 3 with 9, producing the two cubes, 1728 and 27, whose roots, 12 and 3, multiplied together give 36, Ans., as before.

10. $\sqrt[3]{1000} = 10$; 79000m. ÷ 10 = 7900m., Ans.

11. 5 ÷ 2 = 2.5in., radius of largest sphere;
$4 : 1 :: 2.5^3 : 3.90625$; $\sqrt[3]{3.90625} = 1.5749$, 1st Ans.
$4 : 2$ or $2 : 1 :: 2.5^3 : 7.8125$; $\sqrt[3]{7.8125} = 1.98425$;
$1.98425 - 1.5749 = .40935$, 2d Ans.
$4 : 3 :: 2.5^3 : 11.71875$; $\sqrt[3]{11.71875} = 2.2714$;
$2.2714 - 1.98425 = .28715$, 3d Ans.
$2.5 - 2.2714 = .2286$, 4th Ans.

A ROOT OF ANY DEGREE.

3. $131621\dot{7}03842267136\dot{6}$(36, Ans.
$3^{11} = 177147$
Trial Div. $= 3^{10} \times 11 = 649539$)11390700, Dividend.
$36^{11} = 131621703842267136$

4. $26\dot{5}748499\dot{5}710348\dot{8}$(222, Ans.
$2^7 = 128$, 1st Subtrahend.
1st Trial Div. $= 2^6 \times 7 = 448$)1377, 1st Dividend.
$22^7 = 2494357888$, 2d Subtrahend.
2d Tr.D. $= 22^6 \times 7 = 793659328$)1631271077, 2d Dividend.
$222^7 = 26574849957103488$, 3d Subtr.

5. $\sqrt{3808955502493696} = 61716736$;
$\sqrt{61716736} = 7856$, Ans.

6. $\sqrt{23045377697175681} = 151807041$;
$\sqrt{151807041} = 12321$; $\sqrt{12321} = 111$, Ans.

7. $\sqrt{233217204680499310881000000} = 15271450641000$;
$\sqrt[3]{15271450641000} = 24810$, Ans.

ARITHMETICAL PROGRESSION.

(ART. **344**, p. 241.)

3. $2 + 20 \times 5 = 102$, Ans.
4. $500 - 45 \times 10 = 50$, Ans.
5. $3 + 14 \times 2 = 31$, 1st Ans.; $3 + 49 \times 2 = 101$, 2d Ans.
6. $\$100 + 30 \times \$6 = \$280$, Ans.

NOTE.—In example 6, and all similar examples, the *principal* is the *first* term, the *amount* for the *first year* is the *second* term, the *amount* for the *second* year is the *third* term, etc.; ∴ *the number of terms is one more than the number of years.*

7. $3\frac{1}{2} + 19 \times 2\frac{1}{2} = 51$m., Ans.
8. $1 + 19 \times 2 = 39$c., Ans.

(ART. **346**, p. 242.)

2. $148 - 27 = 121$; $121 \div 11 = 11$, Ans.
3. $\$2.50 - \$1 = \$1.50$; $1.50 \div 25 = .06$, Ans.
4. $34 - 1 = 33$; $33 \div 11 = 3$yr., Ans.
5. $241\frac{1}{4} - 16\frac{1}{12} = 225\frac{1}{6}$; $225\frac{1}{6} \div 7 = 32\frac{1}{6}$ft., Ans.
7. $37 - 2 = 35$; $35 \div 5 = 7$, common difference;
   9, 16, 23, 30, Ans.
8. $17 - 1 = 16$; $16 \div 8 = 2$, common difference;
   3, 5, 7, 9, 11, 13, 15, Ans.
9. $27 - 3 = 24$; $24 \div 6 = 4$, common difference;
   23, 19, 15, 11, 7, Ans.
11. $18 - 2 = 16$; $16 \div 4 = 4$, common difference;
    2, 6, 10, 14, 18, 22, 26, 30, 34, Ans.
12. $47 - 32 = 15$; $15 \div 5 = 3$, common difference;
    47, 44, 41, 38, 35, 32, 29, 26, 23, 20, 17, 14, 11, 8, 5, 2, Ans.

(ART. **348**, p. 244.)

2. $37 - 7 = 30$; $30 \div 3 = 10$; $10 + 1 = 11$, Ans.
3. $35 - 5 = 30$; $30 \div 2 = 15$; $15 + 1 = 16$, Ans.

4. $29 - 8 = 21$; $21 \div 4\frac{1}{5} = 5$; $5 + 1 = 6$, Ans.

5. $209\frac{1}{12} - 16\frac{1}{12} = 193$; $193 \div 32\frac{1}{6} = 6$; $6 + 1 = 7$, Ans.

(Art. **350**, p. 245.)

3. $\frac{2 + 92}{2} \times 10 = 470$, Ans.

4. $\frac{4 + 28}{2} \times 9 = 144$, Ans.

5. $(1 + 24) \times \frac{24}{2} = 25 \times 12 = 300$, Ans.

6. $(1 + 12) \times \frac{12}{2} = 78$; $78 \times 2 = 156$, Ans.

Note. — A common clock strikes 78 times in 12 hours, and twice 78 times in 24 hours.

7. 10m. = 3200rd.; $3200 + 1 = 3201$, No. of terms;
$1 + 1 = 2$, 1st term, and also common difference;
$3201 + 3201 = 6402$, last term;
$\frac{2 + 6402}{2} \times 3201 = 10249602$rd. = 32030m. 2rd., Ans.

8. $3 + 3 = 6$, 1st term; $3203 + 3203 = 6406$, last term;
$\frac{6 + 6406}{2} \times 3201 = 10262406$rd. = 32070m. 6rd., Ans.

9. $\frac{16\frac{1}{12} + 144\frac{3}{4}}{2} \times 5 = 402\frac{1}{12}$ft., Ans.

(Art. **351**, p. 246.)

2. $20 \times 8 = 160$; $160 + 5 = 165$, last term;
$\frac{5 + 165}{2} \times 21 = 1785$, Ans.

3. $40 \times \frac{1}{2} = 20$; $20 + 1\frac{1}{2} = 21\frac{1}{2}$, last term;
$\frac{1\frac{1}{2} + 21\frac{1}{2}}{2} \times 41 = 471\frac{1}{2}$, Ans.

4. $32\frac{1}{6} \times 5 = 160\frac{5}{6}$; $160\frac{5}{6} + 16\frac{1}{12} = 176\frac{11}{12}$, last term;
$(16\frac{1}{12} + 176\frac{11}{12}) \times \frac{6}{2} = 579$ft., Ans.

(ART. **353**, p. 246.)

2. $196 \div \frac{7}{2} = 56$; $56 - 7 = 49$, **Ans.**
3. $352 \div \frac{11}{2} = 64$; $64 - 57 = 7$, **Ans.**
4. $1029\frac{1}{3} \div \frac{8}{2} = 257\frac{1}{3}$; $257\frac{1}{3} - 241\frac{1}{4} = 16\frac{1}{12}$ft., **Ans.**
5. $200 \div \frac{10}{2} = 40$; \$40 — \$2 = \$38, **Ans.**

GEOMETRICAL PROGRESSION.

(ART. **360**, p. 249.)

3. $(\frac{1}{2})^9 = \frac{1}{512}$; $64 \times \frac{1}{512} = \frac{1}{8}$, Ans.
4. $2^{16} = 65536$; 65536m. = \$65.536, **Ans.**

NOTE.—The first term being 1, the power of the ratio need not be multiplied by it.

5. $1.06^6 = 1.418519112256$; \$1.418519112256, **Ans.**
6. $1900 - 1780 = 120$; $120 \div 12 = 10$;
   $2^{10} = 1024$; \$100 × 1024 = \$102400, Ans.
7. $1975 - 1855 = 120$; $120 \div 12 = 10$;
   $2^{10} = 1024$; \$40000000 × 1024 = \$40960000000, **Ans.**
8. 1000^{10} = 1,000,000,000,000,000,000,000,000,000,000;
   1000)1,000,000,000,000,000,000,000,000,000,000
   2)1,000,000,000,000,000,000,000,000,000pt.
   8)500,000,000,000,000,000,000,000,000qt.
   4)62,500,000,000,000,000,000,000,000pk.
   8)15,625,000,000,000,000,000,000,000bush.
   1,953,125,000,000,000,000,000,000
   \$17,578,125,000,000,000,000,000,000, **Ans.**

Had the price been \$1, the value of the crop would have been as many dollars as there are bushels; but the price being \$1$\frac{1}{8}$, the total value is $\frac{1}{8}$ more dollars than there are bushels.

The teacher, **who can** ***explain orally*** **as he works, will probably** ***abbreviate*** **the above solution as follows:**

2)1000)1,000,000,000,000,000,000,000,000,000,000

8)5

4)62 5

8)15 625

1 953 125

$17,578,125,000,000,000,000,000,000, **Ans.**

(Art. **361**, p. 250.)

$$3.\quad \left.\begin{array}{l} 486 \div 2 = 243;\ \sqrt[5]{243} = 3 \\ \text{or, } 2 \div 486 = \frac{1}{243};\ \sqrt[5]{\frac{1}{243}} = \frac{1}{3} \end{array}\right\}\text{Ans.}$$

5. $256 \div 1 = 256$; $\sqrt[8]{256} = 2$, ratio; $\therefore$
Series $= 1, 2, 4, 8, 16, 32, 64, 128, 256$, Ans.

6. $768 \div 3 = 256$; $\sqrt[4]{256} = 4$, ratio; $\therefore$
Series $= 3, 12, 48, 192, 768, 3072, 12288, 49152, 196608$, Ans.

7. $128 \div 16 = 8$; $\sqrt[3]{8} = 2$, ratio; $\therefore$
Series $= 128, 64, 32, 16, 8, 4, 2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$, **Ans.**

(Art. **363**, p. 251.)

2. $45927 - 7 = 45920$; $45920 \div 2 = 22960$;
$22960 + 45927 = 68887$, Ans.

3. $5120 - 5 = 5115$; $5115 \div 3 = 1705$;
$1705 + 5120 = 6825$, Ans.

(Art. **364**, p. 251.)

2. $6^{12} = 2176782336$

7

15237476352, last term;

7, first term;

ratio less 1 = 5)15237476345, difference of extremes;

3047495269

15237476352, greater extreme;

18284971621, **sum of series, Ans.**

3.
$$\begin{array}{rrl} 1 \times 10^{11} = & 100000000000, & \text{last term;} \\ & 1, & \text{first term;} \\ \hline \text{ratio less } 1 = & 9\overline{)99999999999}, & \text{difference of extremes;} \\ & 11111111111 & \\ & 100000000000, & \text{greater extreme;} \\ \hline & \$111111111111, & \text{Ans.} \end{array}$$

4.
$$\begin{array}{rrl} 1 \times 5^{11} = & 48828125, & \text{last term;} \\ & 1, & \text{first term;} \\ \hline \text{ratio less } 1 = & 4\overline{)48828124}, & \text{difference of extremes;} \\ & 12207031 & \\ & 48828125, & \text{greater extreme;} \\ \hline & \$61035156, & \text{Ans.} \end{array}$$

ANNUITIES.

(ART. 371, p. 253.)

2. $\frac{1+5}{2} \times 5 = 15$; $\$500 \times .06 = \$30$;
$\$30 \times 15 = \$450$; $\$500 \times 6 + \$450 = \$3450$, Ans.

3. $\frac{1+15}{2} \times 15 = 120$; $\$225 \times .015 = \$3.375$;
$\$3.375 \times 120 = \$405$; $\$225 \times 16 + \$405 = \$4005$, Ans.

4. $\frac{1+7}{2} \times 7 = 28$; $\$6000 \times .07 = \$420$;
$\$420 \times 28 = \$11760$; $\$6000 \times 8 + \$11760 = \$59760$, Ans.

5. $\frac{1+6}{2} \times 6$, or $(1+6) \times \frac{6}{2} = 21$; $\$350 \times .04 = \$14$;
$\$14 \times 21 = \$294$; $\$350 \times 7 + \$294 = \$2744$, Ans.

6. $(1+2) \times \frac{2}{2} = 3$; $\$600 \times .10 = \$60$; $\$60 \times 3 = \$180$;
$\$600 \times 3 + \$180 = \$1980$, interest due;
$\$600 \div .10 = \$6000$, principal;
$\$6000 + \$1980 = \$7980$, Ans.

7. $(1 + 2) \times \frac{2}{2} = 3$; \$600 × .06 = \$36; \$36 × 3 = \$108;
\$600 × 3 + \$108 = \$1908, interest due;
\$600 ÷ .06 = \$10000, principal;
\$10000 + \$1908 = \$11908, Ans.

(Art. **372**, p. 254.)

2. 1 × 1.06^5 = 1.3382255776, last term;
1.3382255776 — 1 = .3382255776, difference of extremes;
1.06 — 1 = .06, ratio less 1;
.3382255776 ÷ .06 = 5.63709296;
\$5.63709296 + \$1.3382255776 = \$6.9753185376, amount of \$1;
\$6.9753185376 × 500 = \$3487.6592688, Ans.

3. \$30.840217, amount of \$1 for 17yr. at 7 per cent. in table;
\$30.840217 × 900 = \$27756.1953, Ans.

4. \$33.065954, amount of \$1, found in table;
\$33.065954 × 1000 = \$33065.954, Ans.

5. \$16.869941, amount of \$1 for 12 years, at 6 per cent.;
\$16.869941 × 150 = \$2530.49115, Ans.

6. \$20.023588, amount of \$1 for 15 years, at 4 per cent.;
\$20.023588 × 300 = \$6007.0764, Ans.

7. 4 years = 16 quarters, No. of installments;
1 × 1.015^{15} = 1.25023206+, greater extreme;
1.25023206 — 1 = .25023206, difference of extremes;
1.015 — 1 = .015, ratio less 1;
.25023206 ÷ .015 = 16.68213
1.25023, greater extreme;
\$17.93236, amount of \$1;
\$17.93236 × 225 = \$4034.781, Ans.

8. 1 × 1.03^7 = 1.2298738654+, greater extreme;
1.2298738654 — 1 = .2298738654, difference of extremes;
1.03, — 1 = .03, ratio less 1;
.2298738654 ÷ .03 = 7.66246218
1.22987386, greater extreme;
\$8.89233604, amount of \$1;
\$8.89233604 × 500 = \$4446.168+, Ans.

9. $8 \div 2 = 4$, No. of installments.
$1 \times 1.12^3 = 1.404928$, greater extreme;
$1.404928 - 1 = .404928$, difference of extremes;
$1.12 - 1 = .12$, ratio less 1;
$.404928 \div .12 = 3.3744$;
\$3.3744 + \$1.404928 = \$4.779328, amount of \$1;
\$4.779328 × 10000 = \$47793.28, Ans.

10. \$33.759992 × 300 = \$10127.9976, Ans.

(Art. **373**, p. 258.)

3. \$9.107914, present worth of \$1, found in table;
\$9.107914 × 175 = \$1593.88495, Ans.

4. \$8.863252 × 150 = \$1329.4878, Ans.

5. \$2000 ÷ 16 = \$125, one installment;
\$10.105895 × 125 = \$1263.236875, Ans.

6. \$11.65229 × 500 = \$5826.145, Ans.

(Art. **374**, p. 258.)

2. \$1200 ÷ .06 = \$20000, Ans.

3. \$900 ÷ .03 = \$30000, Ans.

(Art. **375**, p. 259.)

2. \$1000 ÷ .06 = \$16666.666+, present worth if in possession;
\$16666.666 ÷ 1.1236 = \$14833.274+, Ans.

3. \$13.590322 × 300 = \$4077.0966, present worth for 20yr.;
\$6.732745 × 300 = \$2019.8235, present worth for 8yr.;
\$2057.2731, Ans.

(Art. **376**, p. 259.)

2. \$6000 ÷ 7.721735 = \$777.027+, Ans.

PERMUTATIONS, ARRANGEMENTS, AND COMBINATIONS.

(ART. **379**, p. 261.)

2. As 1 to $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 \times 13 \times 14 \times 15 \times 16 \times 17 \times 18 \times 19 \times 20$, i.e.

As 1 to 2432902008176640000, Ans.

3. $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 = 479001600$, Ans.

4. $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 = 5040$, Ans.

(ART. **382**, p. 263.)

2. $16 \times 15 \times 14 \times 13 \times 12 \times 11 = 5765760$, Ans.

3. $26 \times 25 \times 24 \times 23 \times 22 \times 21 = 165765600$, Ans.

(ART. **385**, p. 264.)

2. $$\frac{26 \times 25 \times 24 \times 23 \times 22 \times 21}{1 \times 2 \times 3 \times 4 \times 5 \times 6} = 230230, \text{ Ans.}$$

3. $$\frac{7 \times 6 \times 5}{1 \times 2 \times 3} = 35, \text{ Ans.}$$

4. The numerator will be the product of the numbers from 50 down to 18, inclusive, and the denominator will be the product of the numbers from 1 up to 33, inclusive; the answer is 9847379391150.

5. $$\frac{56 \times 55}{1 \times 2} = 1540, \text{ Ans.}$$

6. $$\frac{20 \times 19 \times 18 \times 17 \times 16 \times 15 \times 14 \times 13 \times 12 \times 11}{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10} =$$
$18475.60, Ans.

MISCELLANEOUS EXAMPLES.

1. The 1st number is 1 time the 1st;
The 2d number is $3\frac{1}{2}$ times the 1st;
The 3d number is $2\frac{1}{4} \times 3\frac{1}{2} = 7\frac{7}{8}$ times the 1st;
$\therefore$ 99 is $12\frac{3}{8}$ times the 1st;

$$\text{and}\quad \left.\begin{array}{r} 99 \div 12\frac{3}{8} = 8,\ \text{1st} \\ 8 \times 3\frac{1}{2} = 28,\ \text{2d} \\ 8 \times 7\frac{7}{8} = 63,\ \text{3d} \end{array}\right\} \text{Ans.}$$

2.

	yr.	m.	d.
	1858	7	4
	1856	5	12
Time,	2	1	22

SIMPLE INTEREST.

$356.50
.128⅔
11883
11883
285200
71300
35650
6)$45.86966, Int. at 6 per ct.
$7.64494, Int. at 1 per ct.
7½
382247
5351458
$57.33705, Ans.

COMPOUND INTEREST.

$356.50
1.07½
17825
249550
35650
$383.238
1.07½
191619
2682666
383238
$411.981, Amt. for 2yr.
.008⅔
137327
137327
3295848
6)$3.570502, Int. at 6 per ct.
$.595083, Int. at 1 per ct.
7½
297541
4165581
$4.463122, Int. at 7½ per ct.
(carried back)

COMPOUND INTEREST (*continued*).

$4.463122, Int. at 7½ per ct.
$411.981
$416.444, Amount.
$356.50 , Principal.
$59.944, Ans.

3. $\frac{\cancel{7}}{\cancel{13}} \times \frac{\overset{3}{\cancel{81}}}{\cancel{11}} \times \frac{\overset{3}{\cancel{39}}}{\cancel{7}} \times \frac{\cancel{11}}{\cancel{27}} = 9$, Ans.

4. The number multiplied by *half* itself gives $4\frac{1}{2}$, $\therefore$ the number multiplied by *itself* gives 2 times $4\frac{1}{2} = 9$; but a number multiplied by *itself* gives the *square of the number*. Hence, $\sqrt{9} = 3$, Ans.

5. If the number multiplied by *one third* of itself gives $8\frac{1}{3}$, then the number multiplied by *itself* gives 3 times $8\frac{1}{3} = 25$; $\therefore$ 25 is the *square* of the number, and $\therefore$ the number is $\sqrt{25} = 5$, Ans.

6. The number multiplied by *itself* gives $55 \div 2\frac{1}{5} = 25$; $\therefore$ the number is $\sqrt{25} = 5$, Ans.

7. $5280 \times 20\frac{1}{2} = 108240$, No. of feet in $20\frac{1}{2}$ miles;
$$\left.\begin{array}{l} 108240 \div 10.5 = 10308\frac{4}{7}, \text{ hind wheel} \\ 108240 \div 9 = 12026\frac{2}{3}, \text{ fore wheel} \end{array}\right\} \text{Ans.}$$

8. $365 \times 4 + 1 = 1461$, No. of days in 4 years;
\$25000 × 4 = \$100000, salary for 4 years;
\$100000 — \$26950 = \$73050, sum spent;
\$73050 ÷ 1461 = \$50, Ans.

9.
$$\left.\begin{array}{lrll} & \text{lb.} & & \\ & 14 & \text{at 10c. are worth} & \$1.40 \\ & 14 & \text{at 12c. are worth} & \$1.68 \\ & \underline{28} & \text{at 15c. are worth} & \underline{\$4.20} \\ \therefore \text{the} & 56 & \text{are worth} & \$7.28 \\ \text{and} & 1 & \text{is worth } \$7.28 \div 56 = 13\text{c.} & \end{array}\right\} \begin{array}{c}\text{Alligation} \\ \text{Medial.}\end{array}$$

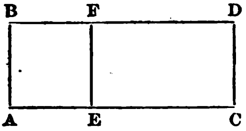
$$19\text{c.}\left\{\begin{array}{l} \text{c.} \\ 13 \\ 18 \\ 23 \end{array}\right. \quad \begin{array}{r} \text{lb.} \\ 4 \\ 4 \\ 6+1=7 \end{array} \quad \left.\begin{array}{l} 4 : 56 :: 4 : 56\text{lb. at 18c.} \\ 4 : 56 :: 7 : 98\text{lb. at 23c.} \end{array}\right\} \text{Ans.}$$

10. This is an example in Alligation, Prob. 3, p. 200. First find how many cubic inches the crown would measure if it were made of pure gold, then how many if made of pure silver. Then, by the linking process, determine the *proportional parts* of silver and gold used in making the crown, and, finally, by Prob. 3, find the number of ounces of *silver used*, and this will evidently be the number of ounces of *gold purloined;* thus,

63 ÷ 10.36 = 6.08108108c. in. of gold;
63 ÷ 5.85 = 10.76923076c. in. of silver;

$$8.2245 \left\{ \begin{matrix} 6.08108108 \\ 10.76923076 \end{matrix} \right] \quad \begin{matrix} 2.54473076, \text{ gold;} \\ \underline{2.14341892}, \text{ silver;} \end{matrix}$$

Sum of proportional parts 4.6881496
4.6881496 : 63 :: 2.1434189 : 28.80355oz., Ans.



11. If $\frac{1}{3}$ of the 30-acre lot ACDB is cut off by the line EF perpendicular to AC, then AEFB is a *square* containing 10 acres = 1600 square rods; ∴

$$\left. \begin{matrix} AB = \sqrt{1600} = 40\text{rd., breadth} \\ \text{and } AC = 3 \text{ times } 40 = 120\text{rd., length} \end{matrix} \right\} \text{Ans.}$$

12. By the conditions of the example the son is to have twice as much as the mother, and the mother twice as much as the daughter; ∴ if the daughter has 1 part the mother will have 2 parts and the son 4 parts; i.e. the estate is divided into 7 parts, and ∴ the mother has $\frac{2}{7}$ of the estate. But if there had been no son the mother would have had $\frac{2}{3}$ of the estate; ∴ $\frac{2}{3} - \frac{2}{7}$ of the estate = \$3200; $\frac{2}{3} - \frac{2}{7} = \frac{8}{21}$; now if $\frac{8}{21}$ = \$3200, $\frac{1}{21}$ = \$400, and ∴ $\frac{7}{21}$ (the mother's share if there had been no daughter) = 7 times \$400 = \$2800, Ans.

13. Each daughter has 1 part, the mother 2 parts, and the son 4 parts; i.e. the estate is divided into 8 parts, and the mother ∴ has $\frac{2}{8}$ or $\frac{1}{4}$ part of the estate. Hence $\frac{2}{3} - \frac{1}{4}$ of the estate, or $\frac{5}{12}$ of the estate = \$3200, and ∴ $\frac{1}{12}$ = \$640 and $\frac{4}{12}$ (the mother's share if there had been only a son) = 4 times \$640 = \$2560, Ans.

14. 8567£ 12s. 3d. + 3567£ = 12134£ 12s. 3d.;
30983£ 14s. 6d. — 12134£ 12s. 3d. = 18849£ 2s. 3d.;
$\frac{1}{3}$ of $\frac{133338}{502643}$ of 18849£ 2s. 3d. = 1666£ 14s. 6d., one daughter's share;
1666£ 14s. 6d. × 3 = 5000£ 3s. 6d., three daughters' shares;
18849£ 2s. 3d. — 5000£ 3s. 6d. = 13848£ 18s. 9d.;
13848£ 18s. 9d. ÷ 5 = 2769£ 15s. 9d., one son's share.

15. The carriage costs $\$20 \div 100 = 20$c. per mile, and each man pays only his share of the expense incurred while he is riding; i.e. when only 2 men ride each pays half of 20c. = 10c. per mile, when 3 men ride each pays $\frac{1}{3}$ of 20c. = $6\frac{2}{3}$c., etc.; $\therefore$ the cost for the several distances and for each man is as follows:

	Cost.	1st man.	2d man.	A.	B.	C.
For 20m.	$4	$2.00	$2.00			
" 30m.	6	2.00	2.00	$2.00		
" 35m.	7	1.75	1.75	1.75	$1.75	
" 15m.	3	.60	.60	.60	.60	$0.60
" 100m.	$20 =	$6.35 +	$6.35 +	$4.35 +	$2.35 +	$0.60, Ans.

16. $(116 + 129 - 1) \div 2 = 122$, number of men in one side of the 1st square; $122^2 + 116 = 15000$, Ans.

EXPLANATION.—The 116 men left by the first arrangement and the 129 men required to complete the second arrangement together constitute one line of men on each of two sides of the second square; then taking out the corner man, and dividing the remainder, 244, by 2, the quotient, 122, is the number of men in each side of the first square; therefore the square of 122 plus the 116 men remaining constitute the army. So, if there were 122 men in one side of the first square, there would manifestly be 123 men in one side of the second square, and $\therefore$ the square of 123 *minus* the 129 men wanting constitute the army; i.e.

$$123^2 - 129 = 15000, \text{ Ans., as before.}$$

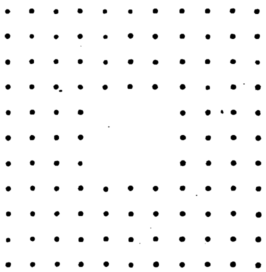
17. If only 1 four-inch pipe were used, and the quantity of water were the same, the answer would be found (Art. 330, Prop. 5) by the following proportion:

$$4^2 : 6^2 :: 4\text{h.} : \text{Ans.};$$

but as 3 four-inch pipes are used, the time by that condition is only $\frac{1}{3}$ as great, and as twice the quantity of water is discharged, the time by *that* condition is twice as great; hence the following proportion:

$$3 \times 4^2 : 2 \times 6^2 :: 4\text{h.} : 6\text{h., Ans.}$$

18. The dots in the annexed diagram representing the arrangement of the men, it will be seen that if the *outer* rank of men around the square consists of a certain number, the next rank consists of 8 men less, the next of 8 less, and so on. Now if the last rank were formed upon the *outside* of the square it would consist of the *excess* in one case, 114, plus the *deficiency* in the other, 114 = 228; ∴ in an arrangement of 4 deep,



the outer rank contains	228 men
the 2d rank contains	220 men
the 3d rank contains	212 men
the 4th rank contains	204 men
∴ the 4 ranks contain	864 men
Subtract the deficiency,	114 men
And there remain	750 men, 1st Ans.

But if the last rank were formed upon the *inside* of the square in an arrangement of 4 deep,

the inner rank contains	228 men
the 2d rank contains	236 men
the 3d rank contains	244 men
the 4th rank contains	252 men
∴ the 4 ranks contain	960 men
Subtract the deficiency,	114 men
And there remain	846 men, 2d Ans.

If the successive ranks are formed on two sides *within* and on the other two sides *without* the square, it will be found, by constructing a diagram, that each succeding rank requires precisely the same number of men; ∴ the *inner* + the *outer rank*, when so formed, will consist of *twice* 228 = 456 men, and also the other two ranks of *twice* 228 = 456 men;

∴ the 4 ranks contain	912 men
Subtract the deficiency,	114 men
And there remain	798 men, 3d Ans.

Note. — The ranks cannot be formed by placing the men on *one* side *within* and on the other *three* sides *without* the square; nor by placing them on *three* sides *within* and on *one* side *without*, for, in either case, *the hollow would not be a square.*

19. 80 : 125 :: $4 : $6.25, Ans.

20. $9.60 ÷ 12 = 80c., cost per pair;
66⅔ : 125 :: 80c. : $1.50, Ans.

21.

$$\begin{array}{r} 2\underline{)2800} \\ 2\underline{)1400} \\ 2\underline{)700} \\ 2\underline{)350} \\ 5\underline{)175} \\ 5\underline{)35} \\ 7 \end{array}$$

Ans. 2, 2, 2, 2, 5, 5, 7.

22. 1st series = 1, 2, 4, 8, 16
2d series = 1, 7,
1st product = 1, 2, 4, 8, 16, 7, 14, 28, 56, 112
3d series = 1, 5, 25

Product of 1st, 2d, and 3d series = 1, 2, 4, 8, 16, 7, 14, 28, 56, 112, 5, 10, 20, 40, 80, 35, 70, 140, 280, 560, 25, 50, 100, 200, 400, 175, 350, 700, 1400, 2800

23.

$$\begin{array}{rr} & 1781 \\ 1027 & \underline{1027} \\ \underline{754} & 754 \\ 273 & \underline{546} \\ \underline{208} & 208 \\ 65 & \underline{195} \\ \underline{65} & 13,\ \text{Ans.} \\ 0 & \end{array}$$

Note. — See note on page 15 of the key.

24. *Debit.*

1st bill due Aug. 30;
2d bill due Oct. 13, 44 days from Aug. 30;
3d bill due Oct. 24, 55 days from Aug. 30.

$$\begin{array}{l}
\$200 \\
400 \times 44 = 17600 \\
\underline{400} \times 55 = \underline{22000} \\
\$1000 \qquad\qquad 39600
\end{array}$$

$39600 \div 1000 = 39\frac{6}{10}$;
40 days from Aug. 30 gives Oct. 9, equated time of debit side.

Credit.

1st bill due Sept. 22;
2d bill due Nov. 29, 68 days from Sept. 22.

$$\begin{array}{l}
\$500 \\
\underline{400} \times 68 = 27200 \\
\$900
\end{array}$$

$27200 \div 900 = 30\frac{2}{9}$;
30 days from Sept. 22 gives Oct. 22, equated time of credit side.

$1000 — $900 = $100, balance due;
From Oct. 9 to Oct. 22 = 13 days;
$900 \times 13 = 11700$; $11700 \div 100 = 117$, No. of days to be reckoned *backward* from Oct. 9, which gives June 14, 1857, Ans.

25. *Debit.*

2d bill (first payable) due May 28;
1st bill due July 8, 41 days from May 28.

$$\begin{array}{l}
\$100 \\
\underline{1500} \times 41 = 61500 \\
\$1600
\end{array}$$

$61500 \div 1600 = 38\frac{7}{16}$;
38 days from May 28 gives July 5, equated time of debit side.

Credit.

1st bill due April 3;
2d bill due May 5, 32 days from April 3.

$$\begin{array}{l}
\$900 \\
\underline{200} \times 32 = 6400 \\
\$1100
\end{array}$$

$6400 \div 1100 = 5\frac{9}{11}$;
6 days from April 3 gives April 9, equated time of credit side.

$1600 — $1100 = $500, balance due;
From April 9 to July 5 = 87 days;
$1100 \times 87 = 95700$; $95700 \div 500 = 191+$, No. of days to be reckoned *forward* from July 5, 1857, which gives Jan. 12, 1858, Ans.

26. Time = 21yr. 10d.;

$$\begin{array}{rl} \$3.207135, & \text{amount of \$1 for 20yr.} \\ 1.06, & \text{amount of \$1 for 1yr.} \\ \hline 19242810 & \\ 3207135 & \\ \hline 3.3995631, & \text{amount of \$1 for 21yr.} \\ 1.001\tfrac{2}{3}, & \text{amount of \$1 for 10d.} \\ \hline 11331877 & \\ 11331877 & \\ 33995631 & \\ 33995631 & \\ \hline \$3.4052290385, & \text{amount of \$1 for 21yr. 10d.} \end{array}$$

$\$3.405229 \times 356 = \$1212.261+$, Ans.

27. $(40 + 30) \times 2 = 140$, distance round the house;
$30 \div 2 + 1 = 16$, base of triangle for finding length of rafter;
$\sqrt{16^2 + 12^2} = \sqrt{400} = 20$, length of rafter.

$$\begin{array}{rl} 20 \times 40 \times 2 = 1600, & \text{area of both sides of roof;} \\ 32 \times 6 \times 2 = 384, & \text{area of two gable ends;} \\ 140 \times 20 = 2800, & \text{area of four walls;} \\ \hline 4784, & \text{Ans.} \end{array}$$

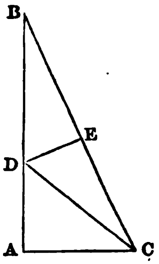
28. $(50 + 30) \times 2 = 160$, length of 4 walls.
$160 \times 20 \times 1 = 3200$, No. of cubic feet.
$3200 \times 27 = 86400$, Ans.

29. $(50 + 29) \times 2 = 158$, length of 4 walls;
$158 \times 20 \times 1\frac{1}{2} = 4740$, No. of cubic feet;
$4740 \times 27 = 127980$, Ans.

30. $\sqrt{9} \times \sqrt{16^2} = 3 \times 16 = 48$, Ans.

31. $\sqrt{\frac{9}{25} \times (\frac{12}{125})^2} = \frac{3}{5} \times \frac{12}{125} = \frac{36}{625}$; $\sqrt{\frac{36}{625}} = \frac{6}{25}$, Ans.

32. $(8^2 - 4^2) \times 3.141592 = 48 \times 3.141592 =$
150.796416sq. in., Ans.



33. From the nature of the case DB = DC, ∴ (Art. 330, Prop. 1) DE, drawn perpendicular to BC, *bisects* BC. The two triangles ABC and DBE, being right-angled at A and E, and having the angle B common, are similar, and their homologous sides are proportional (Art. 330, Prop. 8 and 9).

$BC = \sqrt{AB^2 + AC^2} = \sqrt{32^2 + 16^2} = \sqrt{1280}$, and ∴

$BE = \frac{1}{2}\sqrt{1280}$;

AB : EB :: CB : DB, i.e.

$32 : \frac{1}{2}\sqrt{1280} :: \sqrt{1280} : DB$;

$\frac{1}{2}$ of $\sqrt{1280} \times \sqrt{1280} = \frac{1}{2}$ of $1280 = 640$;

$640 \div 32 = 20 = DB$;

AB — DB = AD, i.e. 32 — 20 = 12ft., Ans.

NOTE. — The square root of any number multiplied by the square root of the same number will produce the number itself; thus $\sqrt{4} \times \sqrt{4} = 2 \times 2 = 4$; $\sqrt{9} \times \sqrt{9} = 9$; and for the same reason, $\sqrt{1280} \times \sqrt{1280} = 1280$; and in any proportion the product of the means divided by either extreme gives the other extreme.

34. $\frac{3}{7}$ of the time past being equal to $\frac{1}{11}$ of the time to come, $\frac{1}{7}$ of the time past equals $\frac{1}{3}$ of $\frac{1}{11} = \frac{1}{33}$ of the time to come, and $\frac{7}{7}$ or the whole of the time past equals 7 times $\frac{1}{33} = \frac{7}{33}$ of the time to come; but the time to come equals $\frac{33}{33}$ of itself; ∴ the whole time from noon to midnight equals $\frac{7}{33} + \frac{33}{33} = \frac{40}{33}$ of the time to come before midnight; i.e. the 12 hours from noon to midnight are to be divided into 40 equal parts, and 7 of those parts are already past.

Now $\frac{7}{40}$ of 12 hours $= \frac{21}{10}$h. $= 2\frac{1}{10}$h. = 2h. 6m., i.e. 6m. past 2, Ans.

35. 1,000,000,000,000,000 ÷ 180 = 5,555,555,555,555$\frac{5}{9}$m.;
5,555,555,555,555$\frac{5}{9}$ ÷ 60 = 92,592,592,592h. 35$\frac{5}{9}$m.;
92,592,592,592 ÷ 24 = 3,858,024,691d. 8h.;
3,858,024,691 ÷ 365$\frac{1}{4}$ = 10,562,695yr. 342$\frac{1}{4}$d.;
∴ reducing the $\frac{1}{4}$ day to hours, and adding it to the 8h., we have 10,562,695 J. yr. 342d. 14h. 35$\frac{5}{9}$m., 1st Ans.

A quadrillion by the English method of numeration is 1,000,000,000 times as many as by the French method; ∴ it will take 1,000,000,000 times as long to count it;

1,000,000,000 times 10,562,695 J. yr. 342d. 14h. 35$\frac{5}{9}$m. are 10,562,695,938,009,650 J. yr. 28d. 20h. 35$\frac{5}{9}$m., 2d Ans.

36. The sum received was $\frac{105}{100}$ of the invoice price; ∴

$12600 ÷ 1.05 = $12000, invoice price, 1st Ans.

$12600 × .01$\frac{1}{2}$ = $189, commission on sale;

$12600 — $189 = $12411, sum left in my hands after taking my pay for selling the goods;

Now, as I am to have 1 per cent. on the purchase, the $12411 is $\frac{101}{100}$ of the sum invested; ∴

$12411 ÷ 1.01 = 12288.11\frac{89}{101}$, 2d Ans.

37. $\frac{18 \times 17 \times 16 \times 15 \times 14 \times 13 \times 12}{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7} = 31824$, 1st Ans. (Art. 385).

$\frac{18 \times 17 \times 16 \times 15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8}{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11} =$ 31824, 2d Ans. (Art. 387).

38. $46 \times 12 = 552$, No. of terms; $2 =$ ratio; $1 =$ 1st term; ∴ (Art. 360),

$2^{551} \times 1 = 2^{551}$, last term; and (Art. 363),

$(2^{551} - 1) \div 1 + 2^{551} = 2 \times 2^{551} - 1 =$

$2^{552} - 1$, sum of the series, i.e. the number of cents the house cost, Ans.

Remark. — The skillful teacher will see in the 38th Ex. a valuable exercise in the construction and combination of formulas, while no one but a tyrant or a dunce will ever require a pupil to *expand* the formulas.

39. $(1 + 552) \times \frac{552}{2} = 152628$c. = $1526.28, Ans.

40. I paid 50 cents *more* than half the money I had, and had $50 remaining after paying the 3d debt; ∴ $50.50 was half and $101 was all the money I had before paying the 3d debt. ∴ By similar reasoning $101.50 was half and $203 was all I had before paying the 2d debt; and $203.50 was half and $407 was all I had at first, Ans.

41. The discharging pipe empties the difference between $\frac{1}{9}$ and $\frac{1}{12}$, viz. $\frac{1}{36}$ of the cistern in an hour; $\therefore$ it would take 36 hours to empty the cistern.

42. $\frac{1}{5}$ of $\frac{3\frac{1}{4}}{5\frac{1}{2}} = \frac{1}{5} \times \frac{13}{4} \times \frac{2}{11} = \frac{13}{110}$;

$\frac{3}{2}$ of $\frac{2}{16\frac{446}{529}} = \frac{3}{2} \times \frac{2}{1} \times \frac{529}{8910} = \frac{529}{2970}$;

$\frac{13}{110} + \frac{529}{2970} = \frac{351}{2970} + \frac{529}{2970} = \frac{880}{2970} = \frac{8}{27}$;

$\sqrt[3]{\frac{8}{27}} = \frac{2}{3}$; $(\frac{2}{3})^2 = \frac{4}{9}$, Ans.

43. $\frac{3}{8}$ of $\frac{5\frac{3}{5}}{4\frac{2}{3}} = \frac{3}{8} \times \frac{28}{5} \times \frac{3}{14} = \frac{9}{20}$;

$\frac{1}{3}$ of $\frac{7\frac{1}{2}}{450} = \frac{1}{3} \times \frac{15}{2} \times \frac{1}{450} = \frac{1}{180}$;

$\frac{9}{20} - \frac{1}{180} = \frac{81}{180} - \frac{1}{180} = \frac{80}{180} = \frac{4}{9}$;

$\sqrt{\frac{4}{9}} = \frac{2}{3}$; $(\frac{2}{3})^3 = \frac{8}{27}$, Ans.

44. $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 =$ 3628800, 1st Ans.

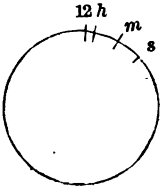
Or, excluding those numbers whose left-hand figure is 0, $3628800 - \frac{1}{10}$ of $3628800 = 3265920$, 2d Ans.

NOTE.—It is evident that $\frac{1}{10}$ of the permutations of the 10 Arabic digits would have 0 for the left-hand figure, for there are only 10 things permuted, and *each* will stand at the left $\frac{1}{10}$ of the time.

45. $\$10000 \div .96 = \$10416.66\frac{2}{3}$ to Liverpool;
$\$10416.666+ \div .99 = \$10521.88552+$ to Havre;
$\$10521.88552 \div .90 = \$11690.9839+$ to San Francisco;
$\$11690.9839 \div .92 = \$12707.59+$, Ans.

46. $36 \times 3 = 108$; $108 \div 4 = 27$; $27 + 11 = 38$;
$38 - 5 = 33$, Ans.

47 (2). If *h*, *m*, and *s* severally represent the positions of the hour, minute, and second hands, then the distance

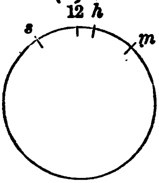


From 12 to *h* = 1 space
From 12 to *m* = 12 spaces
From 12 round and to *s* = 720 spaces

∴ from *h* to *m* = 11 spaces
and from *m* to *s* = 11 spaces
and from 12 to *s* = 23 spaces
and from 12 round to 12 = 697 spaces

Hence, 697 : 1 :: 43200sec. : $61\frac{683}{697}$sec., Ans.

47 (3). Distance from 12 to *h* = 1 space
from 12 to *m* = 12 spaces
from 12 round to *s* = 720 spaces
∴ from *h* to *m* = 11 spaces
and from *s* to *h* = 11 spaces
and from *s* to 12 = 10 spaces
and from 12 round to 12 = 730 spaces



Hence, 730 : 1 :: 43200sec. : $59\frac{13}{73}$sec., Ans.

47 (Remark). To find when the second-hand will be at *s'*, Fig. 1, p. 271, Arithmetic.

Distance from 12 to *h* = 1 space
from 12 to *m* = 12 spaces
from 12 round and to *s'* = 720 spaces
∴ from *h* to *m* = 11 spaces
and from *h* to *s'* = $5\frac{1}{2}$ spaces
and from 12 to *s'* = $6\frac{1}{2}$ spaces
and from 12 round to 12 = $713\frac{1}{2}$ spaces

Hence, $713\frac{1}{2}$: 1 :: 43200sec. : $60\frac{780}{1427}$sec., Ans.

REMARK.—The 49th Ex. may have *three* Solutions. While the h. h. (hour-hand) is between 5 and 6 the m. h. (minute-hand) may fulfil the required conditions, and be between 6 and 7, or between 12 and 1, or it may be *with* the h. h. between 5 and 6.

The two hands cannot make equal angles with the line from 12 to 6 while the h. h. is between 5 and 6 and the m. h. between 11 and 12; for the h. h. is *at* 6 when the m. h. is *at* 12, and however small a distance the m. h. is before 12 the h. h. is only *one twelfth* as far before 6.

49. 1st SOLUTION.—See Fig. 2, p. 272, Arithmetic. At half past 5 o'clock the m. h. is at 6 and the h. h. at *r*, half way from 5 to 6. If the h. h. would remain at rest while the m. h.

moved $2\frac{1}{2}$ minutes, to *l*, the hands would have the required positions; but, while the m. h. is advancing, the h. h. goes from *r* to *s*; ∴ the m. h. must stop at *t*, as much short of *l* as *s* is in advance of *r*; i.e. the hour and minute hands together move over the space represented by $2\frac{1}{2}$ minutes on the dial; but the hour and minute hands together move over 13 spaces while the m. h. alone moves over 12 spaces; hence the proportion:

$$13 : 12 :: 2\tfrac{1}{2}\text{m.} : 2\tfrac{4}{13}\text{m.},$$

the number of minutes beyond half past 5 when the hands will have the required positions, viz. $32\frac{4}{13}$m. past 5 o'clock, Ans.

2d SOLUTION.—At 5 o'clock the m. h. is at 12 and the h. h. at 5. If the h. h. remained at 5 while the m. h. moved from 12 to 1, a space of 5 minutes, the two hands would make equal angles with the line from 12 to 6. But while the m. h. is advancing, the h. h. also advances; ∴ the m. h. must lack as much of going to 1 as the h. h. advances, i.e. the m. h. and h. h. together move over the space represented by 5 minutes. Hence, reasoning as in the preceding case,

$$13 : 12 :: 5\text{m.} : 4\tfrac{8}{13}\text{m.};$$

i.e. the hands will make equal angles with the given line at $4\frac{8}{13}$m. past 5 o'clock, Ans.

3d SOLUTION.—If the circumference of the dial is divided into 12 equal spaces and the h. h. and m. h. start together from 12, the m. h. *gains* 11 spaces upon the h. h. in an hour, but to be with the h. h. again it must gain 12 spaces; ∴ to find when the two hands will be together next after 12 o'clock, we have this proportion:

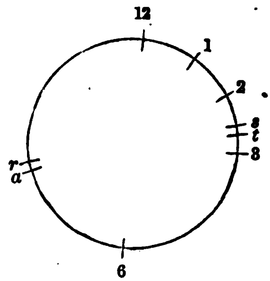
$$11 \text{ spaces} : 12 \text{ spaces} :: 60\text{m.} : 65\tfrac{5}{11}\text{m.} = 1\text{h. } 5\tfrac{5}{11}\text{m.}$$

i.e. the two hands will be together $5\frac{5}{11}$m. past 1 o'clock. But the time required in this solution is the fifth coincidence after 12 o'clock; ∴ the time required is 5 times 1h. $5\frac{5}{11}$m. = 5h. $27\frac{3}{11}$m., i.e. $27\frac{3}{11}$m. past 5 o'clock, Ans.

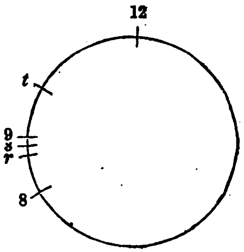
REMARK.—By this plan, multiplying 1h. $5\frac{5}{11}$m. by 2, 3, 4, etc., the times of the successive coincidences of the two hands are easily found.

Reasoning as in the remark on p. 123, it is plain that the h. and m. h. cannot be together between 11 and 12, nor between 12 and 1 o'clock.

50. Reasoning as in the 3d Solution of Ex. 49, we have 1h. 5$\frac{5}{11}$m. × 2$\frac{1}{2}$ = 2h. 43$\frac{7}{11}$m., i.e. 43$\frac{7}{11}$m past 2 o'clock, Ans.



2d Solution. — At half-past 2 the m. h. is at 6 and the h. h. at *s*, half way from 2 to 3. If the h. h. remains at rest the m. h. must move 12$\frac{1}{2}$ minutes, to *a*, in order to point in the opposite direction; but, since the h. h. moves on to *t*, the m. h. must advance to *r*, as much beyond *a* as *t* is in advance of *s*; ∴ since the h. h. moves only $\frac{1}{12}$ as fast as the m. h., the distance from *s* to *t*, or from *a* to *r*, equals $\frac{1}{12}$ of the distance from 6 to *r*, and ∴ the distance from 6 to *a* equals $\frac{11}{12}$ of the distance from 6 to *r*; but the m. h. is 12$\frac{1}{2}$ minutes in going from 6 to *a*, ∴ it is $\frac{12}{11}$ of 12$\frac{1}{2}$m. = 13$\frac{7}{11}$m. in going from 6 to *r*. Hence the required time is 13$\frac{7}{11}$m. beyond half-past 2, viz. 43$\frac{7}{11}$m. past 2, Ans.



51. At a quarter before 9 o'clock the m. h. is at 9 and the h. h. at *r*, $\frac{3}{4}$ of the way from 8 to 9, and while the m. h. moves from 9 to *t*, its position at the time first mentioned in the example, the h. h. moves from *r* to *s*, *its* position at the same moment; ∴ the distance from *r* to *s* equals $\frac{1}{12}$ of the distance from 9 to *t*. Again, at 9 o'clock the h. h. is at 9 and the m. h. at 12, and while the h. h. moves from 9 to *t*, its second required position, the m. h. moves from 12 to *s*, *its* second required position; ∴ the distance from 9 to *t* equals $\frac{1}{12}$ of the distance from 12 to *s*, and ∴ the distance from *r* to *s* equals $\frac{1}{12}$ of $\frac{1}{12}$ = $\frac{1}{144}$ of the distance from 12 to *s* and from 12 to *r* equals the remain-

ing $\frac{143}{144}$ of the distance from 12 to *s*; but the h. h. was $8\frac{3}{4}$ hours in going from 12 to *r*; $\therefore$ if it was $8\frac{3}{4}$ hours in going $\frac{143}{144}$ of the distance from 12 to *s*, the time in which it will go $\frac{144}{144}$, or the whole distance from 12 to *s*, will be found by the proportion:

$$143 : 144 :: 8\tfrac{3}{4}\text{h.} : 8\tfrac{116}{143}\text{h.} = 8\text{h. } 48\tfrac{96}{143}\text{m.},$$

i.e. the first-mentioned time $= 48\frac{96}{143}$m. past 8, Ans.

52. An ox "parteth the hoof and is cloven footed," and consequently he wears 2 shoes upon each foot, and a pair of oxen wear 16 shoes. Hence,
$1 \times 2^{15} = 32768$, last term;
$(32768 - 1) \div (2 - 1) + 32768 = 65535$, sum of series;
65535c. = \$655.35, Ans.

53. The horse "divideth not the hoof," and $\therefore$ he wears only 4 shoes. Hence,
$1 \times 2^{3} = 8$, last term;
$(8 - 1) \div (2 - 1) + 8 = 15$c., Ans.

54. $1 \times 2^{64} =$ 18,446,744,073,709,551,616, last term;
1, first term;
18,446,744,073,709,551,615, dif. of extremes;
18,446,744,073,709,551,616, last term;
\$368,934,881,474,191,032.31, Ans.

NOTE. — Since the ratio is 2, in the above example, it will do no good to divide the difference of the extremes by the ratio less 1.

55. 300d. = 3,000h. = 180,000m. = 10,800,000sec., = No. of dollars counted in 1yr.;
368,934,881,474,191,032 $\div$ 10,800,000 =
34,160,637,173½+yr., Ans.

56. Interest of \$1 for 34,160,637,173½yr. =
6c. $\times$ 34,160,637,173½ = \$2,049,638,230.41; $\therefore$
\$2,049,638,230.41 $\times$ 368,934,881,474,191,032.31 =
\$756,183,037,601,283,999,550,159,534.547, Ans.

57. 34,160,637,173 $\div$ 12 = 2,846,719,764+, No. of terms less 1;
\$368,934,881,474,191,032.31 $\times$ $2^{2846719764}$+, Ans.

MISCELLANY.

(Art. **399**, p. 276.)

1. $$\begin{array}{r} 8)\underline{784800} \\ 98100\text{, Ans.} \end{array}$$

2. $$\begin{array}{r} 8)\underline{9843.28} \\ 1230.41\text{, Ans.} \end{array}$$

3. $$\begin{array}{r} 2)59432700 \\ \underline{29716350} \\ 89149050\text{, Ans.} \end{array}$$

4. $$\begin{array}{r} 2)9864.3 \\ \underline{4932.15} \\ 14796.45\text{, Ans.} \end{array}$$

5. $$\begin{array}{l} 378942 \\ \underline{378942} \\ 3789798942\text{, Ans.} \end{array}$$

6. $$\begin{array}{r} 3569.27 \\ \underline{35.6927} \\ 3604.9627\text{, Ans.} \end{array}$$

7. $$\begin{array}{l} 46923 \\ \underline{328461} \\ 797691\text{, Ans.} \end{array}$$

8. $$\begin{array}{r} 46923 \\ \underline{328461} \\ 3331533\text{, Ans.} \end{array}$$

(Art. **400**, p. 277.)

2. $10 \times 9 + (\frac{1}{2})^2 = 90\frac{1}{4}$, Ans.
3. $13 \times 12 + (\frac{1}{2})^2 = \$1.56\frac{1}{4}$, Ans.

(a). 2. $100 \times 90 + 5^2 = 9025$, Ans.
3. $400 \times 300 + 50^2 = 122500$, Ans.
4. $8000 \times 7000 + 500^2 = 56250000$, Ans.

(Art. **401**, p. 278.)

2. $10 \times 10 - (\frac{1}{2})^2 = 100 - \frac{1}{4} = 99\frac{3}{4}$, Ans.

(b). 2. $4 \times 4 - (\frac{1}{3})^2 = 16 - \frac{1}{9} = 15\frac{8}{9}$, Ans.
5. $60 \times 60 - 2^2 = 3600 - 4 = 3596$, Ans.

(Art. **408**, p. 280.)

2. $869478 \times .06 = 52168.68$, Ans.

(Art. **410**, p. 283.)

2. $$\begin{array}{rcl} \text{19s.} & = & .95£ \\ \underline{\text{8d. 1qr.}} & = & \underline{.034£} \\ \therefore\ \text{19s. 8d. 1qr.} & = & .984£\text{, Ans.} \end{array}$$

3.
$$\begin{array}{rcl} & 12\text{s.} = & .6£ \\ & \underline{11\text{d. }3\text{qr.}} = & \underline{.049£} \\ \therefore & 12\text{s. }11\text{d. }3\text{qr.} = & .649£,\ \text{Ans.} \end{array}$$

(ART. **411**, p. 283.)

2.
$$\begin{array}{rrcl} & .7\ \ £ & = & 14\text{s.} \\ & .05\ £ & = & 1\text{s.} \\ & \underline{.034£} & = & \underline{\quad 8\text{d. }1\text{qr.}} \\ \therefore & .784£ & = & 15\text{s. }8\text{d. }1\text{qr., Ans.} \end{array}$$

3.
$$\begin{array}{rcl} .2\ \ £ & = & 4\text{s.} \\ \underline{.047£} & = & \underline{\quad 11\text{d. }1\text{qr.}} \\ .247£ & = & 4\text{s. }11\text{d. }1\text{qr., Ans.} \end{array}$$

(ART. **412**, p. 283.)

1. 15c. × 300 = \$45.00; \$45.00 ÷ .25 = 180lb., Ans.

or, $\frac{15\text{c.} \times 300}{25} = 180\text{lb.}$, Ans.

2. $\frac{2375 \times 1000}{500} = 4750$, Ans.

3. $\frac{\$4 \times 12}{10} = \$4.80$, Ans.

4. He gives $\frac{23}{25}$ of the price of the coffee, but receives only $\frac{56\frac{1}{4}}{62\frac{1}{2}} = \frac{9}{10}$ of the price of the tea; ∴ he loses the difference, viz. $\frac{23}{25} - \frac{9}{10} = \frac{230}{250} - \frac{225}{250} = \frac{5}{250} = \frac{1}{50} = .02$, i.e. 2 per cent. loss, Ans.

(ART. **417**, p. 285.)

2. $1 \div \frac{1}{2} = 2$, ratio; $1 \div 1 + 1 = 2$, Ans.

3. $\frac{3}{10} \div \frac{3}{100} = 10$, ratio; $\frac{3}{10} \div 9 = \frac{1}{30}$; $\frac{3}{10} + \frac{1}{30} = \frac{1}{3}$, Ans.

(ART. **418**, p. 285.)

2.
$$\begin{array}{lcl} \frac{17 \times (17 + 1) \times (34 + 1)}{6} & = & 1785 \\ \frac{5 \times (5 + 1) \times (10 + 1)}{6} & = & \underline{\quad 55} \\ \text{Difference of series} & = & 1730,\ \text{Ans.} \end{array}$$

(b) 2. $\overline{(12+1)\times 6}^2 = 6084$

$\overline{(4+1)\times 2}^2 = \underline{100}$

Difference of series $= 5984$, Ans. by Rule 1.

$(1+12)\times \frac{12}{2} = 78$; $78^2 = 6084$

$(1+4)\times \frac{4}{2} = 10$; $10^2 = \underline{100}$

Difference of series $= 5984$, Ans. by Rule 2.

CIRCULATING DECIMALS.

2. $.42\dot{1}2\dot{3} = \frac{42}{100} + \frac{123}{99900} = \frac{41958}{99900} + \frac{123}{99900} = \frac{42081}{99900} = \frac{14027}{33300}$, Ans.

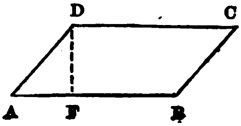
CALENDAR AND CHRONOLOGICAL PROBLEMS.

32. b. Tuesday; d. Sunday.
33. b. Wednesday; d. Monday.
34. b. Wednesday; d. Sunday.
35. b. Sunday; d. Monday.
36. b. Monday; d. Thursday.
37. b. Wednesday; d. Friday.
38. b. Tuesday; d. Saturday.
39. b. Thursday; d. Saturday.
40. b. Tuesday; d. Monday.
41. b. Thursday; d. Monday.
42. b. Sunday; d. Saturday.
43. b. Monday; d. Sunday.
44. b. Friday; d. Sunday.

APPLICATION TO GEOMETRY.

(Art. **465**, p. 317.)

1. $10 \times 4 = 40$sq. in., Ans.
2. $20 \times 7 = 140$sq. rd., Ans.
3. 2ft. = 24in.; $24 \times 3 = 72$sq. in. Ans.
4. In the right-angled triangle AFD, AF = 21 and AD = 35; $\therefore$ FD $= \sqrt{AD^2 - AF^2} = \sqrt{1225 - 441} = \sqrt{784} = 28$; $28 \times 87 = 2436$sq. rd., Ans.

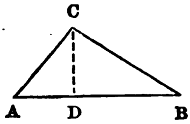


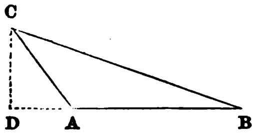
(ART. **466**, **467**, p. 318.)

1. $10 \times \frac{4}{2} = 20$sq. in., Ans. 2. $\frac{20}{2} \times 7 = 70$sq. ft., Ans.

3. 2yd. = 6ft.; $\frac{6}{2} \times 3 = 9$sq. ft., Ans.

4. AD = 28 and AC = 35; ∴ $CD = \sqrt{AC^2 - AD^2} =$





$\sqrt{1225 - 784} = \sqrt{441} = 21$; $\frac{75}{2} \times 21 = 787\frac{1}{2}$sq. ft., Ans.

6. $(7 + 12 + 15) \div 2 = 17$; $17 - 7 = 10$; $17 - 12 = 5$; $17 - 15 = 2$; $\sqrt{17 \times 10 \times 5 \times 2} = \sqrt{1700} = 41.23105$, Ans.

(ART. **468**, **469**, p. 318.)

1. $(7 + 11) \times \frac{4}{2} = 36$sq. ft., Ans.

2. $(10 + 15) \times \frac{6}{2} = 75$, Ans.

4. $24 \div 6 = 4$, length of 1 division;
$(\frac{3}{2} + 6 + 4 + 7 + 5 + 3 + \frac{5}{2}) \times 4 = 116$sq. rd., Ans.

(ART. **470**, **471**, p. 320.)

2. $4 \times 10 \times \frac{5}{2} = 100$sq. ft., Ans.

3. $8 \times 10 \times \frac{12.071068}{2} = 482.84272$, Ans.

5. $4.8284271 \times 5^2 = 120.7106775$sq. rd., Ans.

6. $11.1961524 \times 8^2 = 716.5537536$, Ans.

(ART. **473**, p. 322.)

1. $3.141592 \times 10 = 31.41592$ miles, Ans.

2. $3.141592 \times 25 = 78.5398$ft., Ans.

(ART. **474**, p. 322.)

1. $3.141592 \times 10 = 31.41592$in., length of circumference;
360° : 45° :: 31.41592in. : 3.92699in., Ans.

2. $3.141592 \times 25 = 78.5398$ft., circumference;
360° : 30° :: 78.5398ft. : $6.5449\frac{5}{6}$ft., Ans.

(Art. **475**, p. 322.)

1. $314.1592 \div 3.141592 = 100$in., Ans.
2. $87.964576 \div 3.141592 = 28$b.c., Ans.

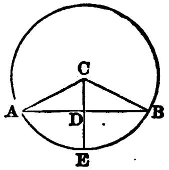
(Art. **477**, p. 324.)

3. $.785398 \times 10^2 = 78.5398$sq. in., Ans. (Rule 2).
4. $.785398 \times 5^2 = 19.63495$sq. rd., Ans.
5. $3.141592 \times 100^2 = 31415.92$sq. m., Ans. (Rule 3).
6. $3.141592 \times 25^2 = 1963.495$sq. yd., Ans.

(Art. **478**, p. 324.)

1. $.785398 \times 10^2 = 78.5398$sq. ft., area of circle;
   360° : 90° :: 78.5398sq. ft. : 19.63495sq. ft., Ans.
2. $3.141592 \times 100^2 = 31415.92$sq. m., circle;
   360° : 120° :: 31415.92sq. m. : $10471.97\frac{1}{3}$sq. m., Ans.

(Art. **479**, p. 324.)



1. Since the arc AEB = 120°, the arc AE or the angle ACD = $\frac{1}{2}$ of 120° = 60°, and ∴ (p. 220, Def. 12), the angle CAD = 30°; hence (p. 223, Prop. 10), CD = $\frac{1}{2}$ AC = 50m., and AD $= \sqrt{AC^2 - CD^2} = \sqrt{10000 - 2500} = \sqrt{7500} = 86.60254$;
   $3.141592 \times 100^2 = 31415.92$sq. m., area of circle;
   360° : 120° :: 31415.92 : $10471.973\frac{1}{3}$, sector;
   $86.60254 \times 50 = 4330.127$, triangle ABC;
   $6141.846\frac{1}{3}$sq. m., Ans.
2. Since the arc AEB = 60°, AE or the angle ACD = 30° and ∴ AD = $\frac{1}{2}$ AC = 50m., and DC $= \sqrt{AC^2 - AD^2} = \sqrt{10000 - 2500} = \sqrt{7500} = 86.60254$;
   Area of circle, as in Ex. 1, equals 31415.92;
   360° : 60° :: 31415.92 : $5235.986\frac{2}{3}$, sector;
   $50 \times 86.60254 = 4330.127$, triangle ABC;
   $905.859\frac{2}{3}$sq. m., Ans.

NOTE.—In solving Ex. 1, Art. 479, supposing the arc AEB is 120°, CD is found to be 50 and AD is 86.60254; and in solving Ex. 2, supposing the arc AEB is 60°, CD is found to be 86.60254 and AD is 50. In each example AD = $\frac{1}{2}$ AB, and $\therefore$ AD × CD, i.e. 86.60254 × 50 in Ex. 1, and 50 × 86.60254 in Ex. 2, gives the area of the triangle ACB = 4330.127. By this we see that if two triangles have their bases and altitudes reciprocally equal, their areas are equal.

(ART. 480, p. 325.)

2. $87.964576 \div 3.141592 = 28$m., diameter;
$.785398 \times 28^2 = 615.752032$, area of circle; $\therefore$
$\sqrt{615.752032} = 24.81435$m., side of square, Ans.

(ART. 481, p. 325.)

3. $40^2 \div 2 = 800$; $\sqrt{800} = 28.28427$in., Ans.

4. $314.1592 \div 3.141592 = 100$, diameter;
$100^2 \div 2 = 5000$; $\sqrt{5000} = 70.71067$, Ans.

(ART. 482, p. 326.)

2. $24 \div 2 = 12$, radius;
$\sqrt{3 \times 12^2} = \sqrt{432} = 20.7846$in., Ans.

3. $87.964576 \div 3.141592 = 28$, diameter;
$28 \div 2 = 14$, radius, also side of hexagon, 1st Ans.
$\sqrt{3 \times 14^2} = \sqrt{588} = 24.24871$, side of triangle, 2d Ans.

(ART. 483, p. 327.)

2. $15 \times 9 \times 3.141592 = 424.11492$, Ans.

3. $12 \times 10 \times 3.141592 = 376.99104$, Ans.

(ART. 484, p. 327.)

1. $60 \times 40 = 2400$sq. in., Ans.

2. $6 \times 10 = 60.$, convex surface;
$2.5980762 \times 2 = 5.1961524$, area of two bases;
65.1961524, Ans.

3. $18 \times 12 = 216$sq. in., Ans.

4. $2 \times 5 \times 3.141592 = 31.41592$, circumference;
$3.141592 \times 5^2 \times 2 = 157.0796$, area of bases;
$31.41592 \times 25 = \underline{785.398}$, convex surface;
942.4776sq. in., Ans.

5. $3.141592 \times 4 = 12.566368$, circumference;

$$\left.\begin{array}{ll} 12.566368 \times 4 & = 50.265472, \text{ convex surface} \\ 3.141592 \times 2^2 \times 2 & = \underline{25.132736}, \text{ bases} \\ \text{Entire surface} & = 75.398208 \end{array}\right\} \text{Ans.}$$

(ART. **485**, p. 329.)

1. $27 \times \frac{18}{2} = 243$sq. in., Ans.

2. $4.8284271 \times 2^2 = 19.3137084$, base;
$2 \times 8 \times \frac{20}{2} = \underline{160.}$, convex surface;
179.3137084sq. ft., Ans.

3. $24 \times \frac{30}{2} = 360$sq. yd., Ans.

4. $3.141592 \times 50 \times 2 = 314.1592$, circumference;
$3.141592 \times 50^2 = 7853.98$, base;
$314.1592 \times \frac{60}{2} = \underline{9424.776}$, convex surface;
17278.756sq. in., Ans.

(ART. **486**, p. 329.)

1. $(5 + 15) \times \frac{6}{2} = 60$sq. ft., Ans.

2. $1.7204774 \times 4^2 = 27.5276384$, lower base;
$1.7204774 \times 2^2 = 6.8819096$, upper base;
$\frac{5 \times 4 + 5 \times 2}{2} \times 7 = \underline{105.}$, convex surface;
Entire surface $= 139.409548$, Ans.

3. $\frac{33 + 27}{2} \times 27 = 810$sq. in., Ans.

4. $314.1592 \div 3.141592 = 100$, diameter of lower base;
$3.141592 \times 40 \times 2 = 251.32736$, circumference of upper base;
$3.141592 \times 50^2 = 7853.98$, area of lower base;
$3.141592 \times 40^2 = 5026.5472$, area of upper base;
$\frac{314.1592 + 251.32736}{2} \times 25 = \underline{7068.582}$, convex surface;
Entire surface $= 19949.1092$, Ans.

(Art. **487**, p. 330.)

1. $3.141592 \times 50 \times 2 = 314.1592$, circumference;
$314.1592 \times 100 = 31415.92$sq. in., Ans.

2. $87.964576 \div 3.141592 = 28$, diameter;
$87.964576 \times 28 = 2463.008128$sq. ft., Ans.

3. $3.141592 \times 4000 \times 2 = 25132.736$, circumference;
$25132.736 \times 8000 = 201061888$sq. m., Ans.

4. $3.141592 \times 896000 = 2814866.432$, circumference;
$2814866.432 \times 896000 = 2522120323072$sq. m., Ans.

(Art. **488**, p. 330.)

1. $3.141592 \times 50 \times 2 = 314.1592$, circumference;
$314.1592 \times 10 = 3141.592$sq. in., Ans.

2. $25132.736 \times 3186 = 80072896.896$sq. m., Ans.
3. $25132.736 \times 2076 = 52175559.936$sq. m., Ans.
4. $25132.736 \times 331 = 8318935.616$sq m., Ans.
South frigid zone $= 8318935.616$sq. m.
South temp. zone $= \underline{52175559.936}$sq. m.
Surface of the earth $= 201061888.000$sq. m. (See Prob. 19, Ex. 3.)

(Art. **489**, p. 331.)

1. 201061888, surface of sphere (See Prob. 19, Ex. 3);
$360° : 36° :: 201061888 : 20106188.8$sq. m., Ans.

2. 2522120323072, surface of sphere (See Prob. 19, Ex. 4);
$360° : 18° :: 2522120323072 : 126106016153.6$, Ans.

(ART. **490**, p. 332.)

2. $3.141592 \times 10 = 31.41592$, circumference;
$31.41592 \times 10 = 314.1592$, surface of sphere;
$314.1592 \div 8 = 39.2699$, $\frac{1}{8}$ of surface;
$100^\circ + 150^\circ + 110^\circ - 180^\circ = 180^\circ$; $\frac{180^\circ}{90^\circ} = 2$;
$39.2699 \times 2 = 78.5398$sq. in., Ans.

(ART. **491**, p. 332.)

2. $15.70796 \div 3.141592 = 5$, diameter;
$15.70796 \times 5 = 78.5398$, surface of sphere;
$78.5398 \div 8 = 9.817475$, $\frac{1}{8}$ of surface;
$150^\circ + 119^\circ\,30' + 75^\circ + 145^\circ + 170^\circ\,30' - 3 \times 180^\circ$
$= 120^\circ$;
$120^\circ \div 90^\circ = \frac{4}{3}$; $9.817475 \times \frac{4}{3} = 13.0899\frac{2}{3}$, Ans.

(ART. **492**, p. 332.)

1. $16 \times 8 = 128$cu. ft., Ans.
2. (See table, p. 321). $6.1818242 \times 3^2 = 55.6364178$, base;
$55.6364178 \times 20 = 1112.7283560$cu. in., Ans.
3. $3.141592 \times 6^2 = 113.097312$, base;
$113.097312 \times 25 = 2827.4328$cu. ft., Ans.
4. $3.141592 \times 5^2 = 78.5398$sq. in., base;
$78.5398 \times 2 \times 12 = 1884.9552$c. in., Ans.

(ART. **493**, p. 333.)

1. $56 \times \frac{27}{3} = 504$c. in., Ans.
2. (See table, p. 321). $11.1961524 \times 10^2 = 1119.61524$, base;
$1119.61524 \times \frac{3 \times 12}{3} = 13435.38288$c. in., Ans.
3. $3.141592 \times 12^2 = 452.389248$, base;
$452.389248 \times \frac{12 \times 12}{3} = 21714.683904$c. in., Ans.
4. $3.141592 \times 8^2 = 201.061888$, base;
$\sqrt{10^2 - 8^2} = \sqrt{36} = 6$, altitude;
$201.061888 \times \frac{6}{3} = 402{,}123776$c. in., Ans.

(ART. **494**, p. 334.)

2. 81, lower base;
36, upper base;
$\sqrt{81 \times 36} = \underline{54}$, mean between the bases;
$171 \times 6 = 1026$cu.ft., Ans.

3. $225 \times 144 = 32400$sq. in., lower base;
$64 \times 144 = 9216$sq. in., upper base;
$\sqrt{32400 \times 9216} = \underline{17280}$sq. in., mean between bases;
$58896 \times 10 = 588960$c. in., Ans.

4. $18.849552 \div 3.141592 = 6$, diameter of upper base;
$3.141592 \times 3^2 = 28.274328$, upper base;
$3.141592 \times 5^2 = 78.5398$, lower base;
$\sqrt{28.274328 \times 78.5398} = \underline{47.12388}$, mean between bases;
$153.938008 \times 5 = 769.69004$,
[Ans.

(ART. **495**, p. 334.)

2. $(12 \times 2 + 15) \times 6 \times 3 = 702$c. in., Ans.

(ART. **496**, p. 335.)

2. $16 \times 18 = 288$, lower base;
$10 \times 12 = 120$, upper base;
$13 \times 15 \times 4 = \underline{780}$, 4 times the parallel section;
$1188 \times 3 \times 12 = 42768$c. in., Ans.

3. $16 \times 18 = 288$, lower base;
$12 \times 10 = 120$, upper base;
$14 \times 14 \times 4 = \underline{784}$, 4 times the parallel section;
$1192 \times 3 \times 12 = 42912$c. in., Ans.

NOTE. — Examples 2 and 3 differ in this: In Ex. 2 the sides that are widest at the lower base are also widest at the upper base; but in Ex. 3 the sides that are widest at the lower base are narrowest at the upper base. This causes a difference in the areas of the parallel sections, and consequently in the cubical contents.

The length and breadth of the parallel sections are found by adding the corresponding dimensions of the lower and upper bases and dividing by 2;

thus, in Ex. 2 the breadths of the lower and upper bases are 16 and 10, ∴ (16 + 10) ÷ 2 = 13, breadth of section parallel to and equidistant from the two bases. So also (18 + 12) ÷ 2 = 15, length of section. In Ex. 3 we have (16 + 12) ÷ 2 = 14, breadth, and (18 + 10) ÷ 2 = 14, length of section.

4. $12 \times 14 = 168$, larger base;
$8 \times 10 = 80$, less base;
$10 \times 12 \times 4 = 480$, 4 times the parallel section;
$\overline{728} \times \frac{40 \times 12}{6} = 58240$c. in., Ans.

(Art. **497,** p. 336.)

1. $3.141592 \times 4000 \times 2 = 25132.736$, circumference;
$25132.736 \times 8000 = 201061888$, surface;
$201061888 \times \frac{4000}{3} = 268082517333\frac{1}{3}$c. m., Ans.

2. (See Art. 487, Ex. 4). 2522120323072, surface;
$2522120323072 \times \frac{896000}{2 \times 3} = 376636634912085333\frac{1}{3}$c. m., Ans.

(Art. **498,** p. 336.)

1. $3.141592 \times 86.60254 = 272.06984684368$, circumference of sphere whose diameter is the altitude of the segment;
$272.06984684368 \times 86.60254 =$
23561.9397940736709472, surface of sphere;
$23561.9397940736709472 \times \frac{86.60254}{2 \times 3} =$
$340087.30558230947519195431 4\frac{2}{3}$c. in. in sphere;
$3.141592 \times 100^2 = 31415.92$, area of larger base;
$3.141592 \times 50^2 = 7853.98$, area of smaller base;
2)39269.90
19634.95, half sum of bases;
$19634.95 \times 86.60254 = 1700436.542773$;
1700436.542773
$340087.305582309475191954314\frac{2}{3}$
$2040523.848355309475191954314\frac{2}{3}$c. in., Ans.

2. $3.141592 \times 13.39746 = 42.08935315632$, circumference of sphere whose diameter is the altitude of the segment;
$42.08935315632 \times 13.39746 = 563.8904253376709472$, surface;
$563.8904253376709472 \times \frac{13.39746}{6} =$ 1259.116569640738834712352cu. ft. in sphere;
$3.141592 \times 50^2 = 7853.98$, base;
$\frac{7853.98}{2} \times 13.39746 = 52611.6914454$;

$$\begin{array}{r} 52611.6914454 \\ 1259.1165696407388834712352 \\ \hline 53870.8080150407388834712352 \end{array}$$

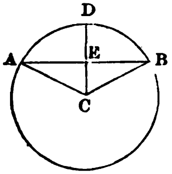
53870.8080150407388834712352cu. ft., Ans.

(Art. **499**, p. 336.)

1. (See Art. 497, Ex. 1). $268082517333\frac{1}{3}$c. m., sphere;
$360° : 36° :: 268082517333\frac{1}{3} : 26808251733\frac{1}{3}$c. m., Ans.
2. (See Art. 497, Ex. 2). $360° : 45° ::$
$37663663491208533\frac{1}{3}$c. in. : $4707957936401066\frac{2}{3}$c. in., [Ans.

(Art. **500**, p. 337.)

1. (See Art. 490, Ex. 1).
$33510314\frac{2}{3} \times \frac{4000}{3} = 44680419555\frac{5}{9}$c. m., Ans.
2. $3.141592 \times 5 = 15.70796$, circumference;
$15.70796 \times 5 = 78.5398$, surface;
$78.5398 \div 8 = 9.817475$, $\frac{1}{8}$ of surface;
$150° + 119°\ 30' + 75° + 145° + 170°\ 30' - 3 \times 180° = 120°$;
$120° \div 90° = \frac{4}{3}$; $9.817475 \times \frac{4}{3} = 13.0899\frac{2}{3}$, base;
$13.0899\frac{2}{3} \times \frac{5}{6} = 10.9083\frac{1}{18}$c. in., Ans.



3. ACB $= 120°$, $\therefore$ ECB $= 60°$ and EBC $= 30°$ (Art. 490, Note). Hence, (Art. 330, Prop. 10) CE $= \frac{1}{2}$ of $12 = 6$, and $\therefore$ ED $= 6$, altitude of the zone which is the base of the spherical sector;

$3.141592 \times 12 \times 2 = 75.398208$, circumference;
$75.398208 \times 6 = 452.389248$, area of base;
$452.389248 \times \frac{12}{3} = 1809.556992$c. in., Ans.

4. $75.398208 \times 12 = 904.778496$, surface of hemisphere;
$904.778496 \times 4 = 3619.113984$c. in. in hemisphere;
$\underline{1809.556992}$c. in. in sector;
1809.556992c. in., Ans.

(Art. **501**, p. 337.)

2. $18.849552 \div 3.141592 = 6$, diameter;
$6^2 \div 3 = 12$; $\sqrt{12^3} = 41.569219$c. in., Ans.

(Art. **502**, p. 338.)

1. $3.141592 \times 5^2 = 78.5398$, base of cylinder;
$78.5398 \times 5 = 392.699$c. in., Ans.

2. The pail is 3 inches greater in diameter at the top than at the bottom, $\therefore$ $\frac{1}{3}$ of the way up it is 1 inch greater, and $\frac{2}{3}$ of the way up it is 2 inches greater than at the bottom; i.e. at the surface of the water before the lobster is immersed the diameter is 11 inches, and after it is immersed the diameter at the surface is 12 inches. Hence, the question is merely to find the contents of the frustum of a cone (Art. 494) whose altitude is 4 inches and the diameters of whose bases are 11 inches and 12 inches.

$3.141592 \times (\frac{11}{2})^2 = 95.033158$, lower base;
$3.141592 \times 6^2 = 113.097312$, upper base;
$\sqrt{95.033158 \times 113.097312} = \underline{103.672536}$, mean between bases;
Ans. $\frac{4}{3} \times 311.803006 = 415.737341\frac{1}{3}$c. in.

Note. — The mean between the bases is evidently $\sqrt{3.141592 \times 6^2} \times \sqrt{3.141592 \times (\frac{11}{2})^2}$, which is the same as the square root of $3.141592^2 \times 6^2 \times (\frac{11}{2})^2 = 3.141592 \times 6 \times \frac{11}{2} = 103.672536$, a process for finding the mean, more simple than that indicated in the above operation.

GAUGING.

2. $(19 - 16) \times .5 + 16 = 17.5$, mean diameter;
$\frac{17.5^2 \times 36}{294} = 37.5$gal., Ans.

3. $(31 - 26) \times .5 + 26 = 28.5$, mean diameter;
$\frac{28.5^2 \times 44}{359} = 99.55153$gal., Ans.

4. $(40 - 32) \times .7 + 32 = 37.6$, mean diameter;
$37.6^2 \times .785398 = 1110.36427648$, mean area;
$\frac{1110.36427648 \times 48}{2150.42} = 24.78468$bush., Ans.

5. $(42 - 36) \times .5 + 36 = 39$, mean diameter;
$\frac{39^2 \times .785398 \times 54}{2150.42} = 29.9978$bush., Ans.

TONNAGE OF VESSELS.

1. $\frac{80 \times 21 \times 18}{95} = 318\tfrac{6}{19}$ tons, Ans.

2. $\frac{200 \times 38 \times 19}{95} = 1520$ tons, Ans.

3. $100 - \tfrac{3}{5}$ of $25 = 85$; $\frac{85 \times 25 \times 20}{95} = 447\tfrac{7}{19}$ tons, Ans.

4. $300 - \tfrac{3}{5}$ of $40 = 276$;
$\frac{276 \times 40 \times 20}{95} = 2324\tfrac{4}{19}$ tons, Ans.

PHILOSOPHICAL PROBLEMS.

(ART. **511**, p. 342.)

1. $1668.75 - 1481.25 = 187.5$; $1668.75 \div 187.5 = 8.9$, Ans.

2. $100 + 300 = 400$, loss; $100 \div 400 = \tfrac{1}{4}$, Ans.

(ART. **512**, p. 342.)

1. $709\tfrac{3}{8} - 645 = 64\tfrac{3}{8}$, loss in sea water;
$709\tfrac{3}{8} - 646\tfrac{7}{8} = 62\tfrac{1}{2}$, loss in distilled water;
$64\tfrac{3}{8} \div 62\tfrac{1}{2} = 1.03$, Ans.

2. $486\frac{1}{4} - 427\frac{1}{2} = 58\frac{3}{4}$, loss in oil;
$486\frac{1}{4} - 423\frac{3}{4} = 62\frac{1}{2}$, loss in water;
$58.75 \div 62.5 = .94$, Ans.

(Art. **515**, p. 344.)

1. $4000 - 1000 = 3000$, distance from center;
$\frac{3000}{4000} = \frac{3}{4}$; $48 \times \frac{3}{4} = 36$lb., Ans.
2. $4000 - 3000 = 1000$; $\frac{1000}{4000} = \frac{1}{4}$;
$8000 \times \frac{1}{4} = 2000$lb., 1st Ans.
$\frac{0}{4000} \times 8000 = 0$, 2d Ans.

(Art. **516**, p. 344.)

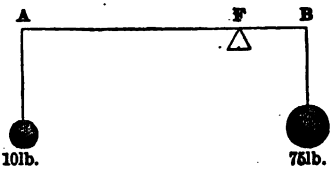
1. $(\frac{4000}{12000})^2 = (\frac{1}{3})^2 = \frac{1}{9}$; $36 \times \frac{1}{9} = 4$lb., Ans.
2. $(\frac{4000}{4005})^2 = (\frac{800}{801})^2 = \frac{640000}{641601}$;
$2000 \times \frac{640000}{641601} = 1995.00935$lb., Ans.

(Art. **518**, p. 345.)

1. $1 : 10^2 :: 16\frac{1}{12} : 1608\frac{1}{3}$ft., Ans.
2. $1 : 20^2 :: 16\frac{1}{12}$ft. : $6433\frac{1}{3}$ft. in 20sec.;
$6433\frac{1}{3} - 1608\frac{1}{3} = 4825$ft., Ans.

(Art. **522**, p. 345.)

1. 2 : 8 :: 100lb. : 400lb., Ans.
2. 6in. : 66in. :: 25lb. : 275lb., Ans.
3. 10.5ft. : 1.5ft. :: 2000lb. : $285\frac{5}{7}$lb., Ans.
4. 31in : 5in. :: 50lb. : $8\frac{2}{31}$lb., Ans.
5. 4000lb. : 200lb. :: 12ft. : $\frac{3}{5}$ft. $= 7\frac{1}{5}$in., Ans.
6. By the principle in Art. 522 we have
75 : 10 :: AF : BF;
and by Art. 439, e, we have
75 + 10 : 10 :: AF + BF : BF;
or reducing, 85 : 10 :: 48in. : BF; ∴



$$BF = \frac{48\text{in.} \times 10}{85} = 5\tfrac{11}{17}\text{in., Ans.}$$

7. In the above figure, suppose BF = 2ft., then
300lb. : 12000lb. :: 2ft. : AF = 80ft., Ans.
8. 40lb. : 550lb. :: 9in. : $123\frac{3}{4}$in., Ans.
9. 6in. : 8ft. = 96in. :: 200lb. : 3200lb., Ans.
10. 8ft. = 96in. : 10in. :: 8000lb. : $833\frac{1}{3}$lb., Ans.
11. 50lb. : 750lb. :: 2ft. : 30ft., Ans.
12. 3000lb. : 150lb. :: 6ft. = 72in. : $3\frac{3}{5}$in., Ans.
13. 20ft. : 5ft. :: 100lb. : 25lb., Ans.
14. 6ft. : 18ft. :: 50lb. : 150lb., Ans.
15. 300lb. : 60lb. :: 30ft. : 6ft., Ans.
16. 100lb. : 500lb. :: 4ft. : 20ft., Ans.

(Art. **524**, p. 347.)

1. 2in. : 3ft. = 36in. :: 50lb. : 900lb., Ans.
2. 3ft. = 36in. : 2in. :: 900lb. : 50lb., Ans.
3. 900lb. : 50lb, :: 3ft. = 36in. : 2in., Ans.
4. 50lb. : 900lb. :: 2in. : 36in. = 3ft., Ans.
5. 12lb. : 160lb. :: 3in. : 40in., radius;
3.141592 × 40 × 2 = 251.32736in. = 20ft. 11.32736in.,
[Ans.

(Art. **528**, p. 348.)

1. 2 : 1 :: 600lb. : 300lb., Ans.
2. 1 : 8 :: 200lb. : 1600lb., Ans.

(Art. **530**, p. 349.)

1. 5ft. : 40ft. :: 200lb. : 1600lb., Ans.
2. 40ft. : 5ft. :: 1600lb. : 200lb., Ans.
3. 1600lb. : 200lb. :: 40ft. : 5ft., Ans.
4. 200lb. : 1600lb. :: 5ft. : 40ft., Ans.
5. 4ft. : 12ft. :: 16lb. : 48lb., Ans.
6. 12ft. : 4ft. :: 48lb. : 16lb., Ans.
7. 16lb. : 48lb. :: 4ft. : 12ft., Ans.
8. 48lb. : 16lb. :: 12ft. : 4ft., Ans.

(ART. **533**, p. 350.)

1. 10ft. = 120in., radius of circle described by the power;
   3.141592 × 120 × 2 = 753.98208in., circumference;
   1in. : 753.98208in. :: 200lb. : 150796.416lb., Ans.
2. 753.98208in. : 1in. :: 150796.416lb. : 200lb., Ans.
3. 200lb. : 150796.416lb. :: 1in. : 753.98208in., circumf.;
   753.98208 ÷ 3.141592 = 240in. = 20ft;
   20ft. ÷ 2 = 10ft., Ans.
4. 150796.416lb. : 200lb. :: 753.98208in. : 1in., Ans.

(ART. **535**, p. 351.)

1. 2in. : 12in. :: 500lb. : 3000lb., pressure on both sides;
   3000lb. ÷ 2 = 1500lb., Ans.
2. 12in. : 2in. :: 6000lb. : 1000lb., Ans.
3. 500lb. : 6000lb. :: 2in. : 24in., Ans.
4. 6000lb. : 500lb. :: 12in. : 1in., half the thickness;
   1in. × 2 = 2in., Ans.
5. Since the edge forms an angle of 60°, the angle formed by the side and a line drawn from the edge to the center of the back is 30°, and ∴ (Art. 330, Prop. 10) half the thickness of the back is half of 15in. = 7.5in.;
   7.5in. : 15in. :: 100lb. : 200lb.; pressure on both sides;
   200lb. ÷ 2 = 100lb., Ans.

(ART. **538**, p. 354.)

1. $1^2 : 4^2$ :: 600lb. : 9600lb., Ans.
2. $1^2 : 12^2$ :: 2190lb. : 315360lb., Ans.
3. 6in. : 2in. :: 86400lb. : 28800lb., Ans.
4. 1.5in. : 3in. :: 39420lb. : 78840lb., Ans.
5. 8 × 3 × 4 : 12 × 2 × 1 :: 19200lb. : 4800lb., Ans.
6. 6 × 3 × 3 : 8 × 6 × 4 :: 59130lb. : 210240lb., 1st Ans.
   6 × 3 × 3 : 8 × 6 × 3 :: 59130lb. : 157680lb., 2d Ans.

7. $\frac{1 \times 1 \times \frac{1}{2}}{1} : \frac{8 \times 2 \times 4}{12}$ :: 600lb. : 6400lb., 1st Ans.

$\frac{1 \times 1 \times \frac{1}{2}}{1} : \frac{8 \times 2 \times 1}{12}$ :: 600lb. : 1600lb., 2d Ans.

8. $\frac{1 \times 1 \times \frac{1}{2}}{1} : \frac{6 \times 3 \times 1\frac{1}{2}}{6}$ :: 2190lb. : 19710lb., 1st Ans.

$\frac{1 \times 1 \times \frac{1}{2}}{1} : \frac{6 \times 3 \times 3}{6}$ :: 2190lb. : 39420lb., 2d Ans.

9. $\frac{1 \times 1 \times \frac{1}{2}}{1} : \frac{6 \times 6 \times 3}{6} \times \frac{3 \times 3}{2 \times 4}$ :: 2190lb. : 88695lb., [Ans.

10. $\frac{1 \times 1 \times \frac{1}{2}}{1} : \frac{10 \times 5 \times 5}{10} \times \frac{5 \times 5}{2 \times 8}$ :: 600lb. : 46875lb., [Ans.

11. $\frac{1 \times 1 \times \frac{1}{2}}{1} : \frac{4 \times 4 \times 2}{4} \times \frac{1}{8}$ :: 600lb. : 1200lb., Ans.

12. $\frac{1 \times 1 \times \frac{1}{2}}{1} : \frac{6 \times 3 \times 3}{6} \times \frac{1}{8}$ :: 2190lb. : 4927.5lb., [Ans.